

# POST-QUANTUM SECURE CRYPTOGRAPHIC IMPLEMENTATIONS FOR EMBEDDED DEVICES

Joppe Bos  
SEPTEMBER 2023



SECURE CONNECTIONS  
FOR A SMARTER WORLD

PUBLIC

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# Agenda

- Who am I?
- Quantum Threat → Post-Quantum → New Standards
- Examples: Applied PQC Innovation
  - PQC Side-Channel Analysis
  - PQC Hardware Re-use
- PQC Use-Cases
  - Low-memory Dilithium
  - PQC in Automotive

Goal: Look at PQC from an industry perspective.  
What research is important and needed?





# SECURE CONNECTIONS FOR A SMARTER WORLD

OUR DIGITALLY ENHANCED WORLD IS EVOLVING TO ANTICIPATE AND AUTOMATE

NXP Semiconductors N.V. (NASDAQ: NXPI) is a global semiconductor company creating solutions that enable secure connections and infrastructure for a smarter world. NXP focuses on research, development and innovation in its target markets.

**AUTOMOTIVE**



**INDUSTRIAL & IOT**

**MOBILE**

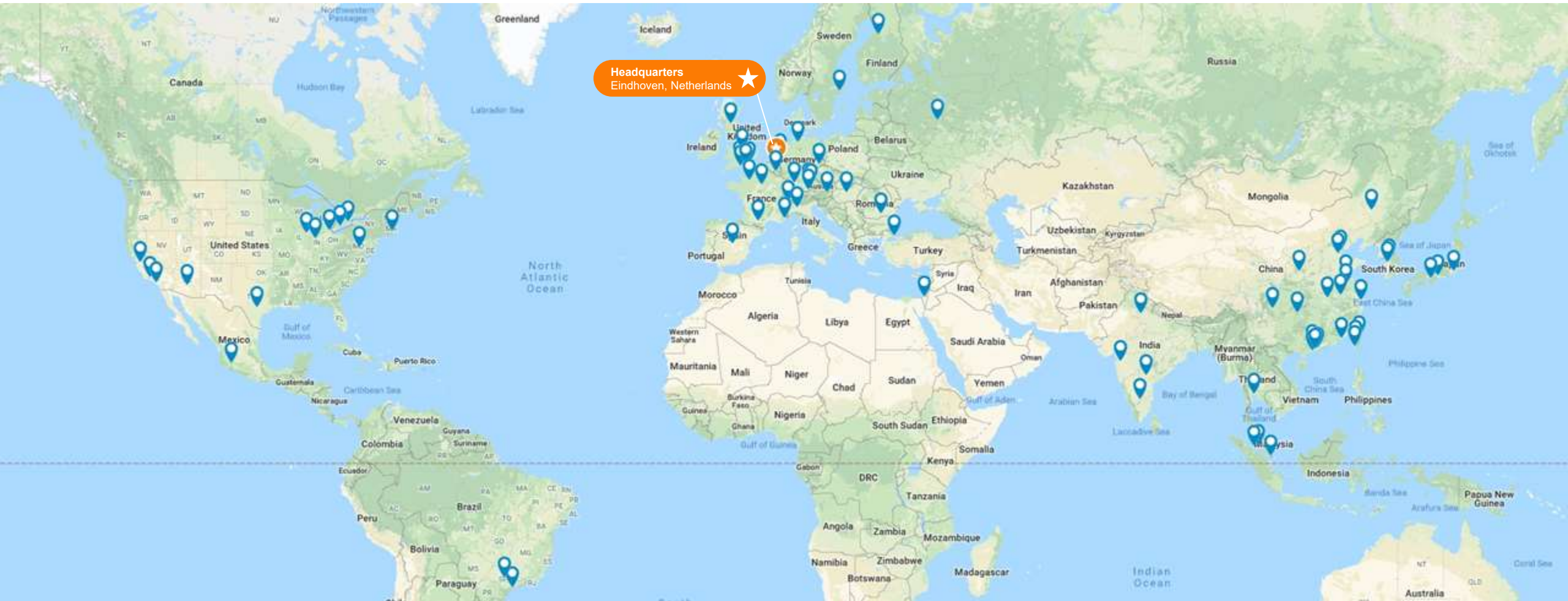


**COMMUNICATION  
INFRASTRUCTURE**



# NXP LOCATIONS

~34,000 employees with operations  
in more than 30 countries



# WHOAMI



## Joppe W. Bos

Cryptographic Researcher and  
Technical Director at NXP  
Semiconductors

Secretary of the IACR (2017-  
2019, 2020-2022)

Editor of the Cryptology ePrint  
Archive (2019-today)

Editor-in-Chief of the IACR  
Communications in Cryptology

## WHOAMI

- Cryptographic Researcher & Technical Director @ NXP
  - Competence Center Crypto & Security in Leuven, Belgium
  - Technical lead of the PQC project
  - Manager of the Crypto Concepts team
  - Head security + crypto funded projects & university relations
- Post-doc
  - Cryptography Research Group at Microsoft Research, Redmond, USA.
- PhD in Cryptology
  - EPFL, Lausanne, Switzerland
- Bachelor / Master in Computer Science
  - University of Amsterdam





## BREAKING ECC

Main PhD project:

112-bit ECDLP solved  
using 224 PlayStation  
3 game consoles.

Bos, Kaihara, Kleinjung, Lenstra,  
Montgomery: Solving a 112-bit Prime  
Elliptic Curve Discrete Logarithm Problem  
on Game Consoles using Sloppy  
Reduction. International Journal of Applied  
Cryptography, 2012.

# QUANTUM THREAT



# POST-QUANTUM

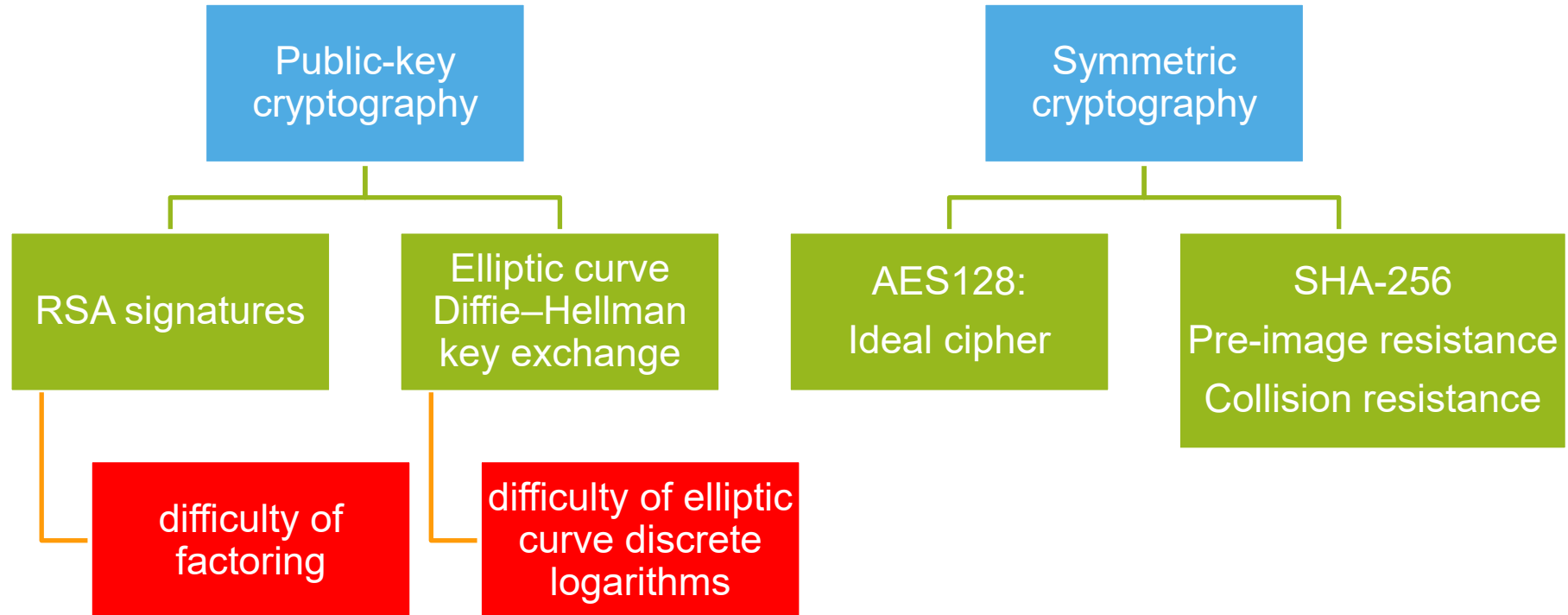


# NEW STANDARDS



# CONTEMPORARY CRYPTOGRAPHY

## TLS - ECDHE - RSA - AES128 - GCM - SHA256



# How IBM's new five-qubit universal quantum computer works

IBM achieves  
CHRIS LEE - 5/4

NEWS | 23 October 2019

## Hello quantum world! Google publishes landmark quantum supremacy claim

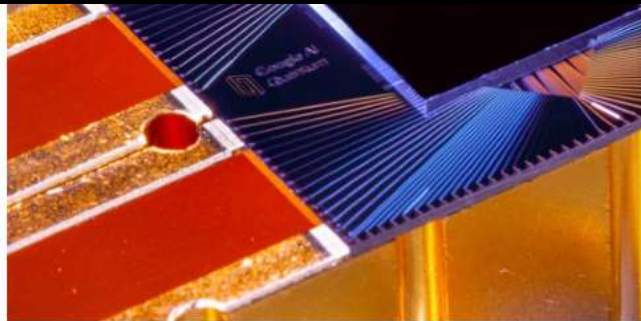
The company says that its quantum computer is the first to perform a task that would be practically impossible for a classical machine.

## Eagle's quantum performance progress

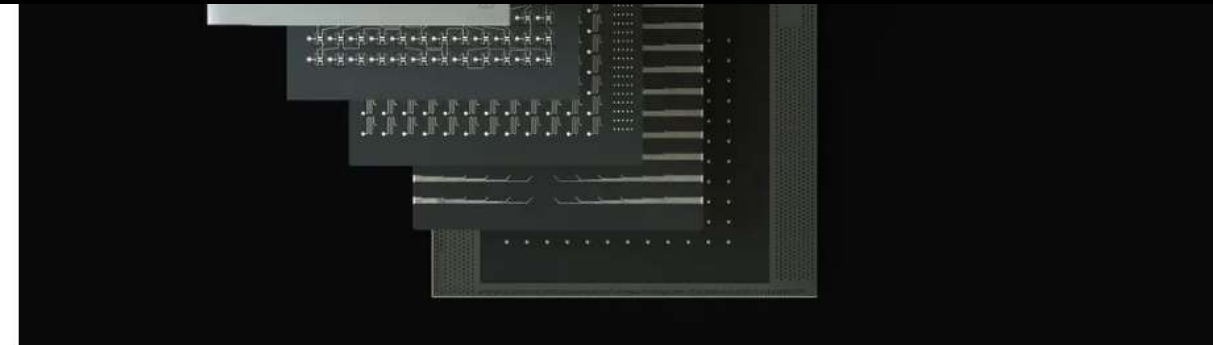
Last November, IBM Quantum announced Eagle, a 127-qubit quantum processor based on the transmon superconducting qubit architecture. The IBM Quantum team adapted advanced semiconductor signal delivery and packaging into a technology

# Intel Delivers 17-Qubit Superconducting Chip with Advanced Packaging to QuTech

ing test chip for quantum Netherlands. The new chip was ved yield and performance.



The Sycamore chip is composed of 54 qubits, each made of superconducting



# QUANTUM COMPUTING

Computer systems and algorithms based on principles of quantum mechanics

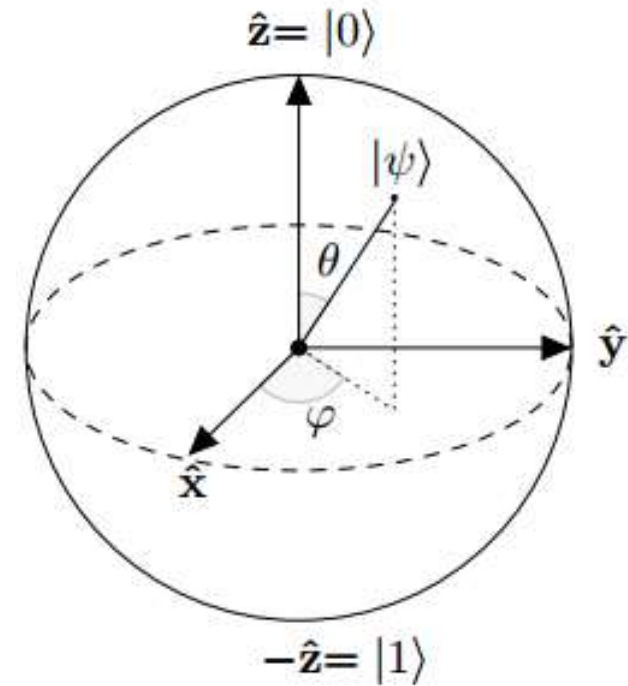
- Superposition
- Interference
- Entanglement

- A classical bit can only be in the state corresponding to 0 or the state corresponding to 1
- A qubit may be in a superposition of both states  
→ when measured it is always 0 or 1

## Shor's quantum algorithm (1994).

Polynomial time algorithm to factor integers.

**Impact.** If we assume the availability of a large quantum computer, then one can break RSA instantly.



## State-of-the-art.

IBM's 127-Qubit Quantum Processor

## Break RSA-3072:

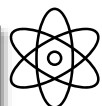
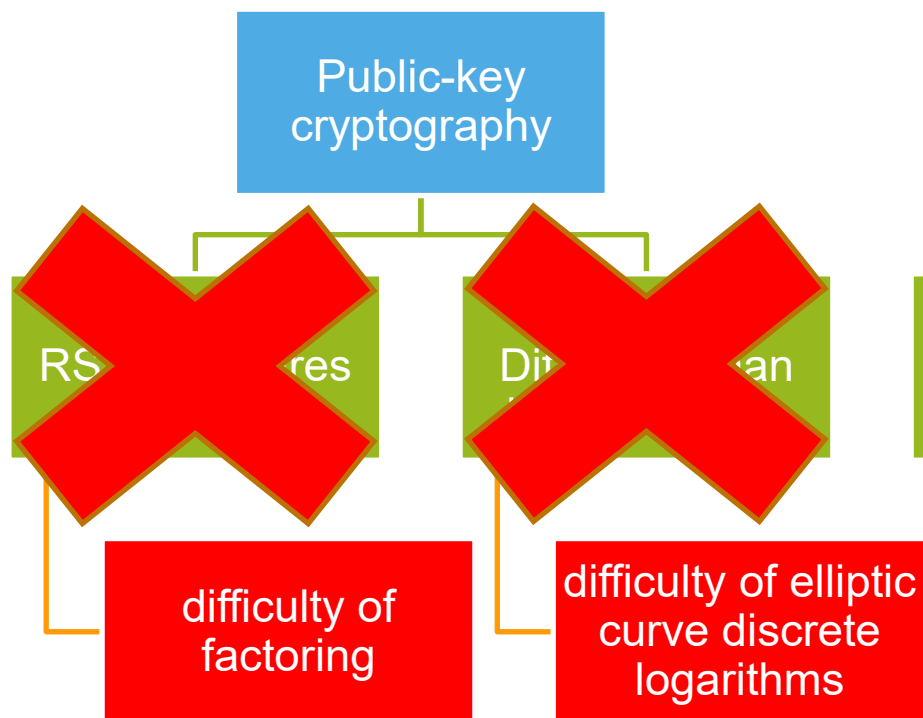
~10,000 qubits are needed



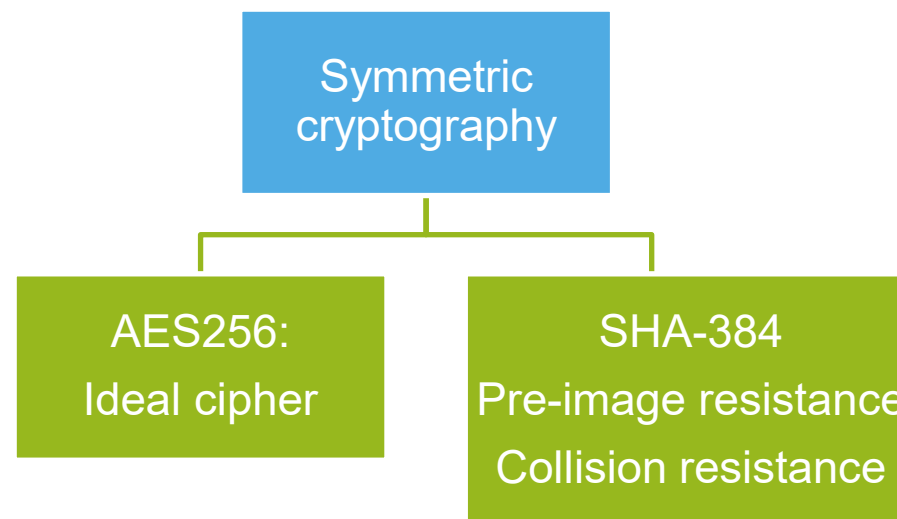
# CONTEMPORARY CRYPTOGRAPHY

TLS - ~~ECDHE~~ - ~~RSA~~ - AES256 - GCM - SHA384

“Double” the key sizes



Shor's algorithm (1994)



Grover's algorithm (1996)

# Quantum Potential to Destroy Security as we know it

## **Confidential email messages, private documents, and financial transactions**

Secure today but could be compromised in the future, even if encrypted

## **Firmware update mechanisms in vehicles**

Could be circumvented and allow dangerous modifications

## **Critical industrial and public service infrastructure (for healthcare, utilities, and transportation using internet and virtual private networks)**

Could become exposed - potentially destabilize cities

## **Audit trails and digitally signed documents associated with safety (auto certification and pharmaceutical authorizations)**

Could be retrospectively modified

## **The integrity of blockchains**

Could be retrospectively compromised - could include fraudulent manipulation of ledger and cryptocurrency transactions







**POST-QUANTUM CRYPTO STANDARDS ARE COMING  
IT DOESN'T MATTER IF YOU BELIEVE IN QUANTUM COMPUTERS OR NOT**



# POST-QUANTUM CRYPTO STANDARDIZATION

**2016**

- Formal call for proposals

**2017**

- Deadline for submissions
- 69 candidates received

**2019**

- Second Round Candidates announced: 26 remaining candidates

**2020**

- Third Round Candidates announced: 7 Finalists and 8 Alternates

**2022**

- **Announcement of Winners to be Standardized**

**2024**

- Standards Available

**2030**

- Migration to new PQC public-key standards completed





## HOW TO PREPARE FOR HURRICANE SEASON

### Quantum



#### MAKE A PLAN

Anten should create an emergency plan and/or checklist

- obtain supplies
- update personal documents
- secure household
- research evacuation options/routes
- update prescriptions



#### CREATE A GO-BAG

Prepare supplies ahead of a hurricane. These can include

- Food/water
- Additional clothes
- Personal documents
- Travel supplies
- Prescriptions



#### KNOW YOUR WING GUIDANCE

Whether preparing for a hurricane or evacuating know your wing or installation's guidance. Routinely check for updates from leadership and maintain communication with your chain of command.



#### RECOGNIZE WARNINGS & ALERTS

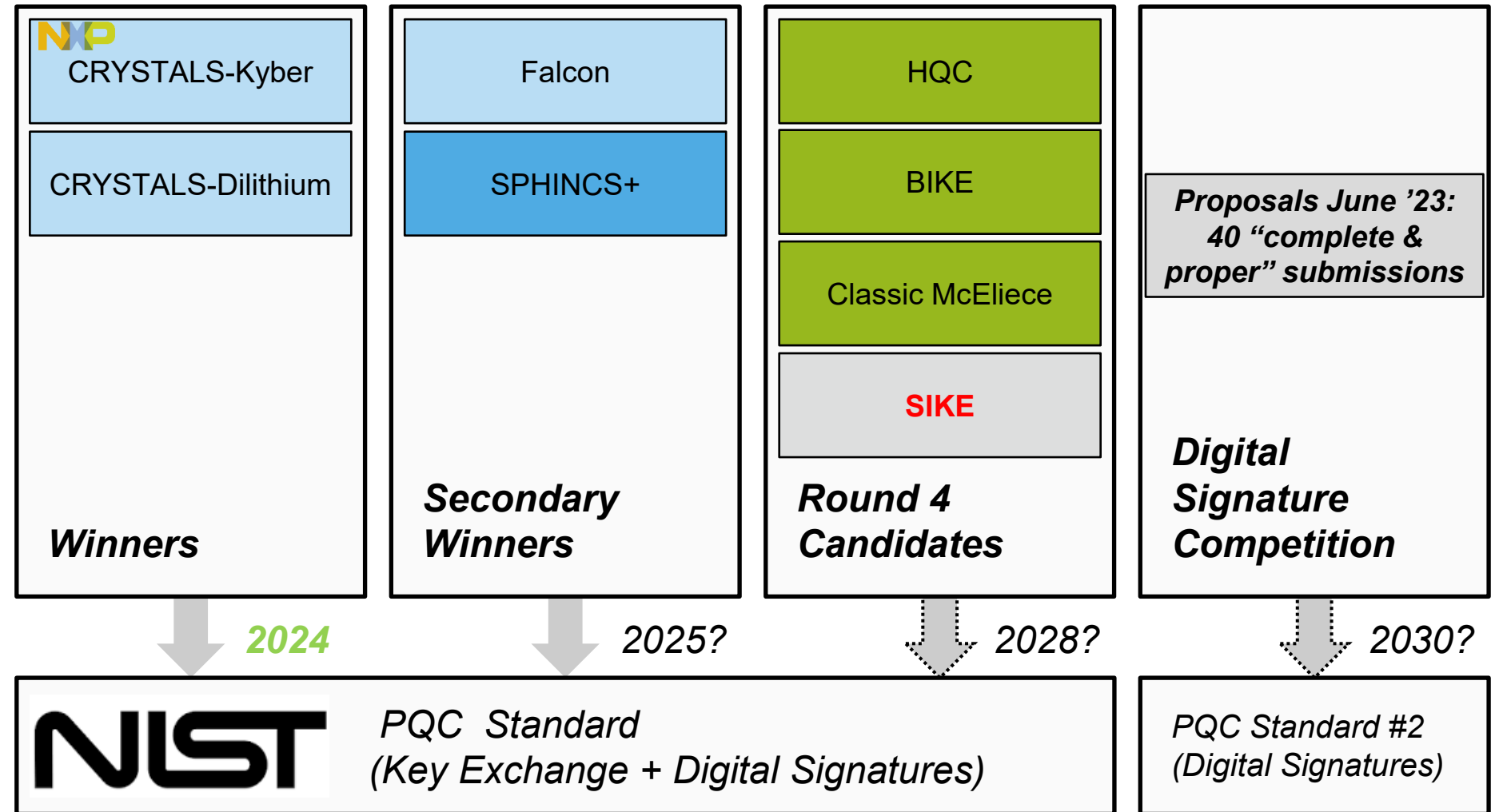
Have several ways to receive alerts. Download real-time alert apps. Sign up for community alerts in your area and be aware of the Emergency Alert System (EAS) and Wireless Emergency Alert (WEA) - which requires no sign up.



#### STAY SAFE

Practice good hygiene and safety measures during any part of a hurricane evacuation or impact. Keep family considerations in mind and don't be afraid to contact leadership for guidance.

## PQC STANDARDS – NIST



Color key: Mathematical approach

Lattice

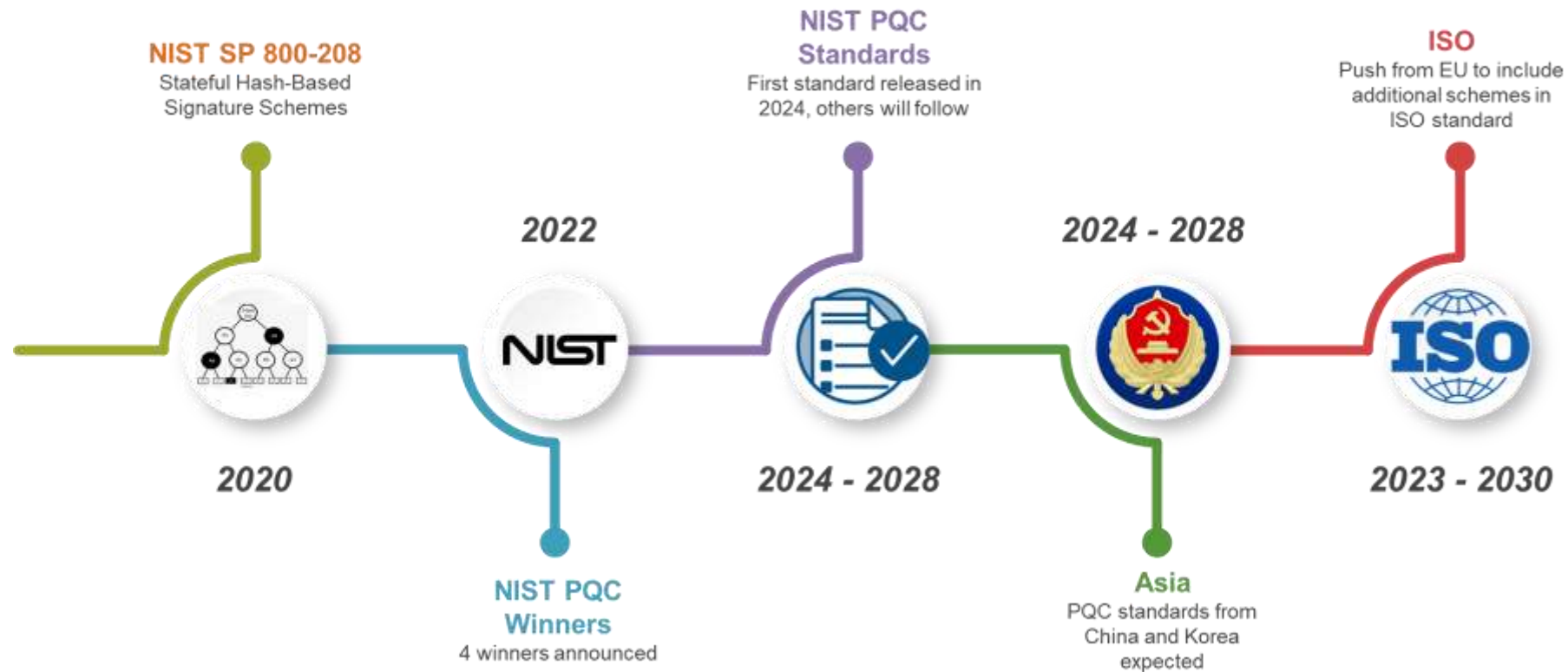
Hash

Code

PUBLIC

15





## National Standards

- **USA.** NIST announces standards release of 4 PQC schemes ('24 – '25). Additional standards to follow.
- **EU.** Push from BSI (help from NXP) for adding schemes to international standard. April '23: ISO to amend [ISO/IEC 18033-2](#).
- **ASIA.** Selection of new schemes ongoing in both China/Korea.

## Protocol Standards

- **IETF:** TLS, OpenPGP, hybrid keys, key serialization, encoding for signatures
- ISO/TC 68/SC 2/WG 11 (Encryption algorithms used in banking applications)
- ISO/IEC JTC1/SC 17/WG 4 (Cards and security devices for personal identification)



# PQC MIGRATION GUIDANCE BY GOVERNMENTS



## USA (NIST/NSA)

- [NIST/NSA recommendation](#) available
  - Commercial National Security Algorithm Suite 2.0
- PQC FW signature recommended for new products after 2025
- PQC transition complete by 2030 using SW update



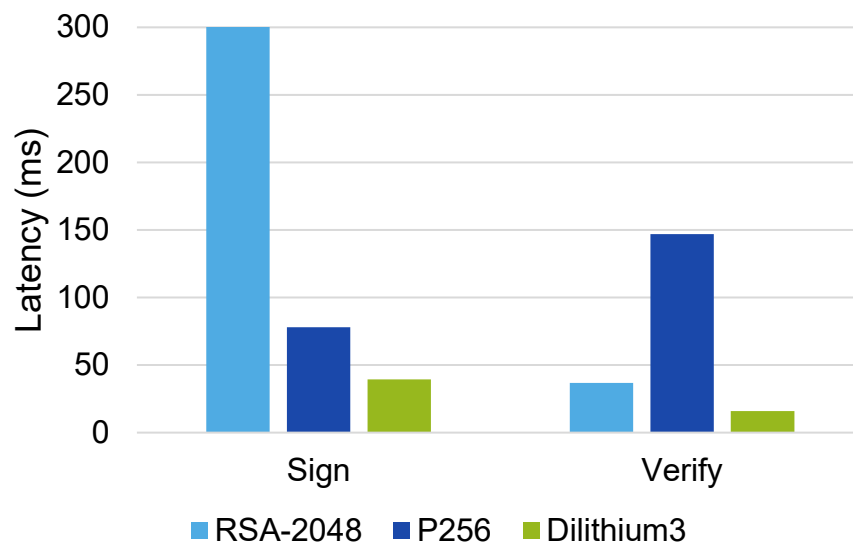
## Germany (BSI)

- [BSI first recommendation](#) (English)
- [BSI considerations](#) (German)
- Expectation is that beginning of 2030s, a relevant quantum computer is available to be a threat for high-secure applications
- Quantum security: considers both PQC + QKD



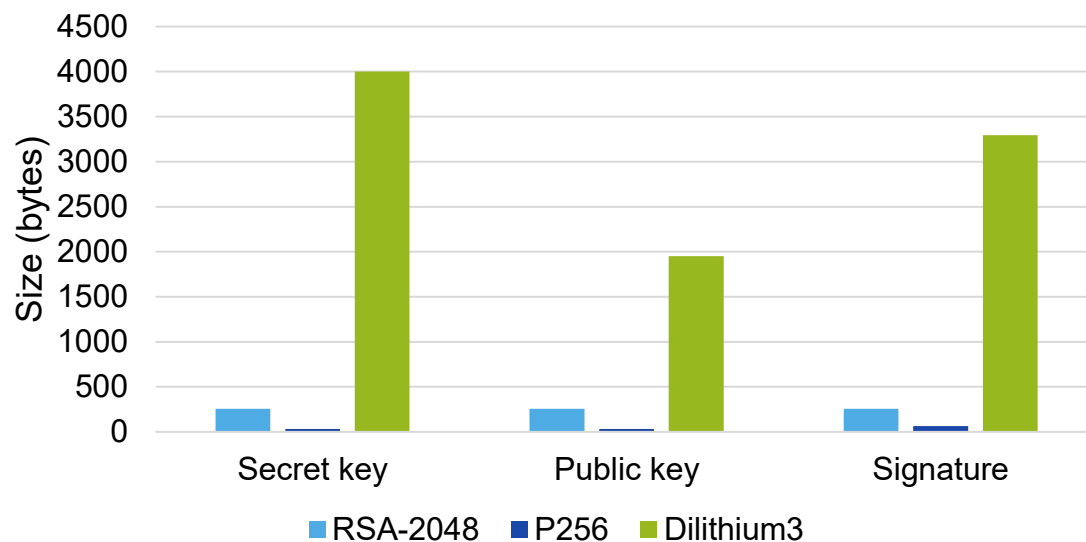
## France (ANSSI)

- PQC for security products “as soon as possible” when long-lasting (until 2030) protection is required
- Others to migrate to classic-PQC hybrid in 2025 – 2030
- Switch to PQC-only expected by 2030



## DILITHIUM IMPACT

- Measurements on Cortex-M4 from pqm4 framework
- Functional implementation only (not hardened)
- Large trade-offs between stack and efficiency
- **80 ~ 90 percent** of run-time in SHA-3



# PQC SIGNATURE MIGRATION (EMBEDDED PERSPECTIVE)

Algorithm (Level 3)	PQ Secure?	Standard?	Efficient Signing?	Stateful?	Efficient Verify?	Need hybrid?	PK (Bytes)	Sig (Bytes)
ECC	No	FIPS 186	Yes	No	Yes	N/A	32 B	64 B
Dilithium	Yes	PQC (2024)	Yes	No	Yes	Yes	1952 B	3293 B
Falcon (L5)	Yes	PQC (2024)	No	No	Yes	Yes	1793 B	1280 B
SPHINCS+	Yes	PQC (2024)	No	No	Yes	No	48 B	16224 B
LMS / XMSS	Yes	SP 800-208	Yes?	Yes	Yes	No	60 B	1744 B



# MODULE LWEE 101

## CRYPTOGRAPHIC SUITE FOR ALGEBRAIC LATTICES (CRYSTALS)

- The Cryptographic Suite for Algebraic Lattices (CRYSTALS) encompasses
  - Kyber – Key Encapsulation Mechanism (KEM)
  - Dilithium – Digital Signatures
- **Theory:** same building blocks
  - Module Learning with Errors
  - Number-Theoretic Transformations



## MODULE (RING) LEARNING WITH ERRORS

$$\begin{array}{c} \text{Lattice} \\ \swarrow \\ \begin{bmatrix} a_{00}(x) & a_{01}(x) \\ a_{10}(x) & a_{11}(x) \end{bmatrix} \\ A \end{array} + \begin{array}{c} \text{Secret key} \\ \swarrow \\ \begin{bmatrix} s_0(x) \\ s_1(x) \end{bmatrix} \\ s \end{array} + \begin{array}{c} \text{"Error"} \\ \swarrow \\ \begin{bmatrix} e_0(x) \\ e_1(x) \end{bmatrix} \\ e \end{array} = \begin{array}{c} \text{Public key} \\ \swarrow \\ \begin{bmatrix} t_0(x) \\ t_1(x) \end{bmatrix} \\ t = As + e \end{array}$$

Given **blue**, find **red** or **yellow**



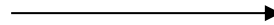
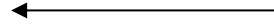
## PUBLIC-KEY ENCRYPTION (DLOG DIFFIE-HELLMAN)



### Exchange

Generate keypair  $(r, u = rP)$

Generate shared secret  $\kappa = rt$



### Key generation

Keypair  $(s, t = sP)$

### Exchange

Compute  $\kappa = us$  (*Diffie-Hellman*)

$$rt = r(sP) = s(rP) = su$$

## PUBLIC-KEY ENCRYPTION (DLOG DIFFIE-HELLMAN + EL GAMAL)



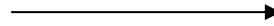
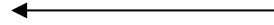
### Encryption

Generate message  $m$

Generate keypair  $(r, u = rP)$

Generate shared secret  $\kappa = rt$

Compute ciphertext  $(u, v) = (u, \kappa + m)$



### Key generation

(Static) Keypair  $(s, t = sP)$

### Decryption

Compute  $\kappa = us$  (Diffie-Hellman)

Recover  $m = v - \kappa$

$$v - \kappa = m + rt - su$$

## PUBLIC-KEY ENCRYPTION (“APPROXIMATE” EL GAMAL)



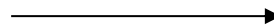
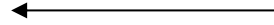
### Encryption

Generate message  $m$

Generate keypair  $(r, u = rA + e_1)$

Generate shared secret  $\kappa = rt$

Compute ciphertext  $(u, v) = (u, \kappa + m + e_2)$



### Key generation

(Static) Keypair  $(s, t = As + e)$



### Decryption

Compute  $\kappa' = us$  (*Diffie-Hellman*)

Recover  $m' = v - \kappa'$

Recover  $m$  from  $m'$

$$v - \kappa' = m + e_2 - e^T r - s^T e_1$$



## PUBLIC-KEY ENCRYPTION (LATTICE-BASED, IND-CPA)



### Encryption

Generate message  $m$

Generate keypair  $(r, u = rA + e_1)$

Generate shared secret  $\kappa = rt$

Compute ciphertext  $(u, v) = (u, \kappa + m + e_2)$

Carefully modify  $u$  (bit flips) and

→ Check whether  $us$  changes

→ Detecting whether decryption succeeds leaks about  $s$

### Key generation

(Static) Keypair  $(s, t = As + e)$



### Decryption

Compute  $\kappa' = us$  (Diffie-Hellman)

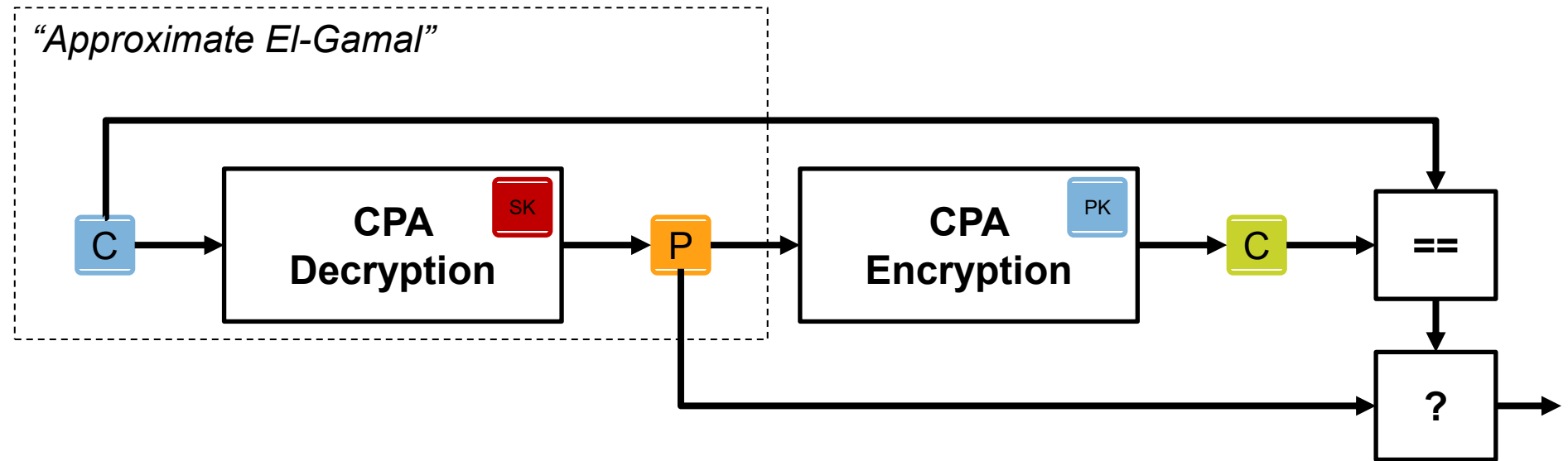
Recover  $m' = v - \kappa'$

Recover  $m$  from  $m'$



Only secure with EPHEMERAL keys

## FUJISAKI OKAMOTO TRANSFORM



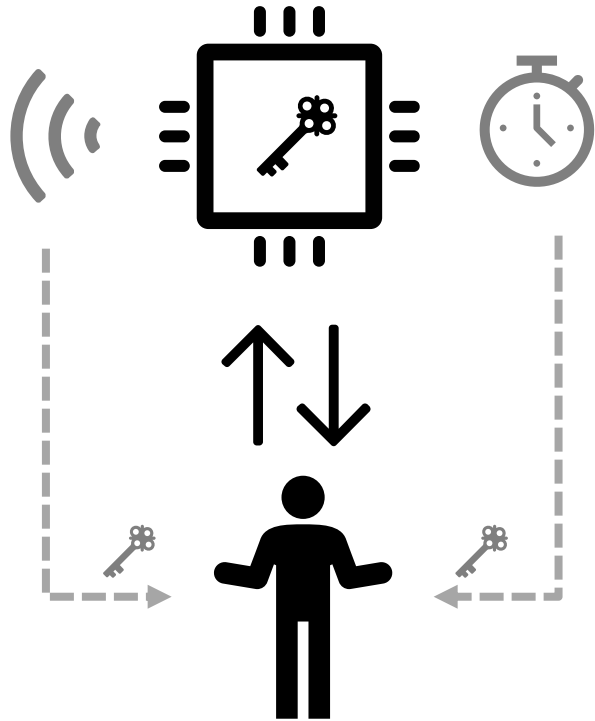
Transform a scheme which achieves **IND-CPA** ("chosen plaintext attack") security to reach **IND-CCA** ("indistinguishability against chosen-ciphertext attacks") security

- Fujisaki, E. and Okamoto T., Secure integration of asymmetric and symmetric encryption schemes, CRYPTO 1999 and JoC 2013

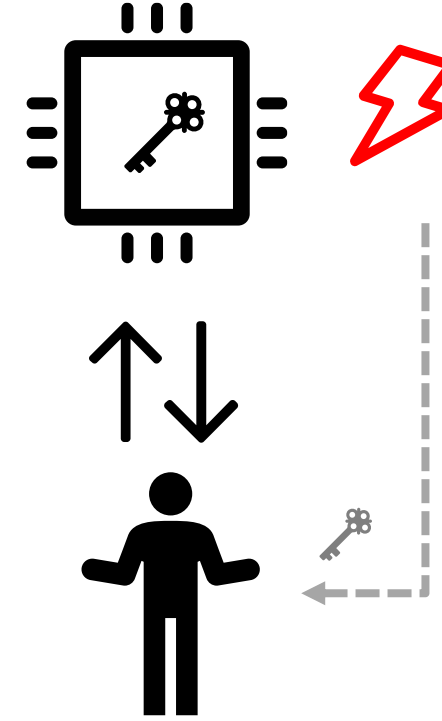
# PQC & SCA



# EMBEDDED CRYPTOGRAPHY AND IMPLEMENTATION ATTACKS



**Side-Channel Attacks (SCA)**



**Fault Attacks (FA)**

# CHALLENGES IN THE EMBEDDED WORLD

## Attacks

Deep understanding in both academia and industry.



## Current Cryptography

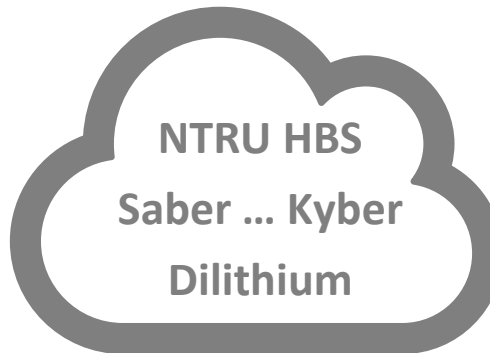


## Countermeasures

Practically secure and certified implementations.

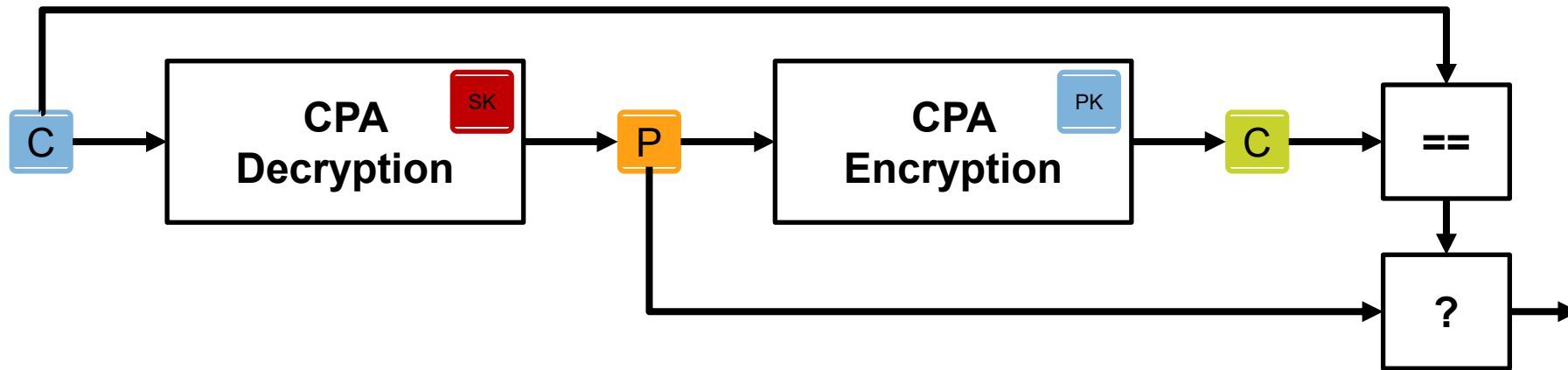
**What does it mean to secure PQC implementations in “practice”?**

Active research area resulting in increasingly powerful attacks.



Early stage of academic research.  
Limited industrial results.

## FUJISAKI OKAMOTO TRANSFORM



Transform a scheme which achieves **IND-CPA** (“chosen plaintext attack”) security to reach **IND-CCA** (“indistinguishability against chosen-ciphertext attacks”) security

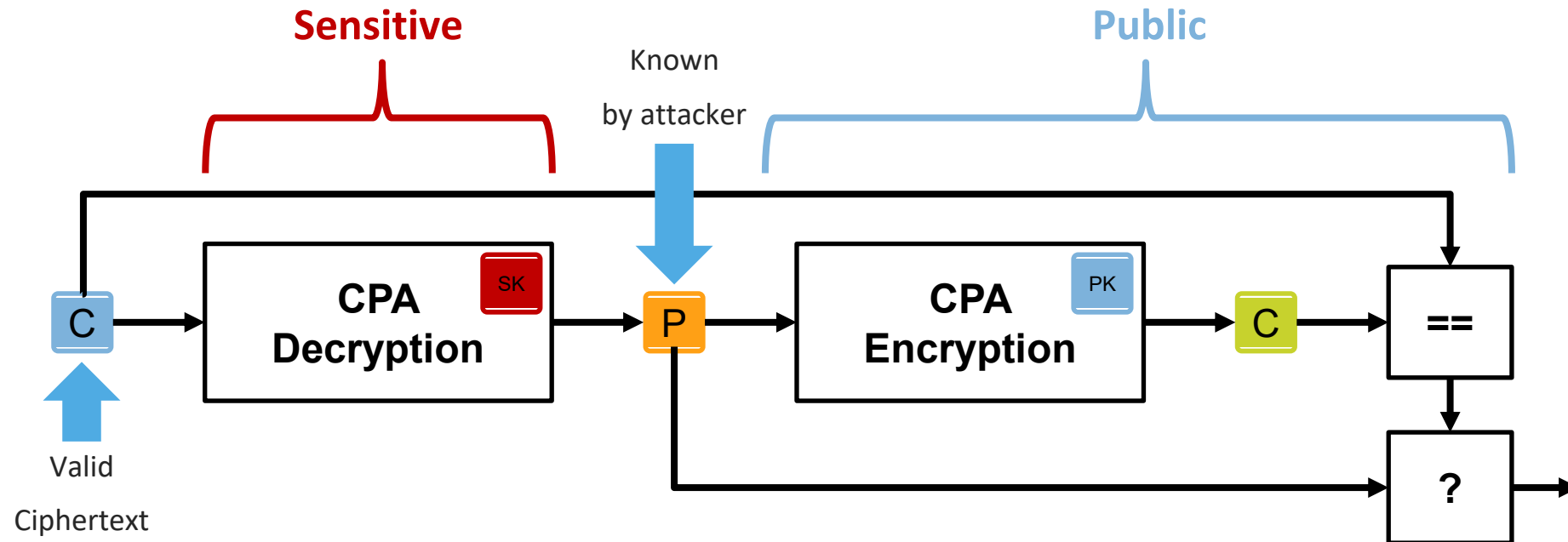
- Fujisaki, E. and Okamoto T., Secure integration of asymmetric and symmetric encryption schemes, CRYPTO 1999 and JoC 2013



# THE SCA PROBLEM OF THE FO-TTRANSFORM

## Attack 1: Chosen Plaintext

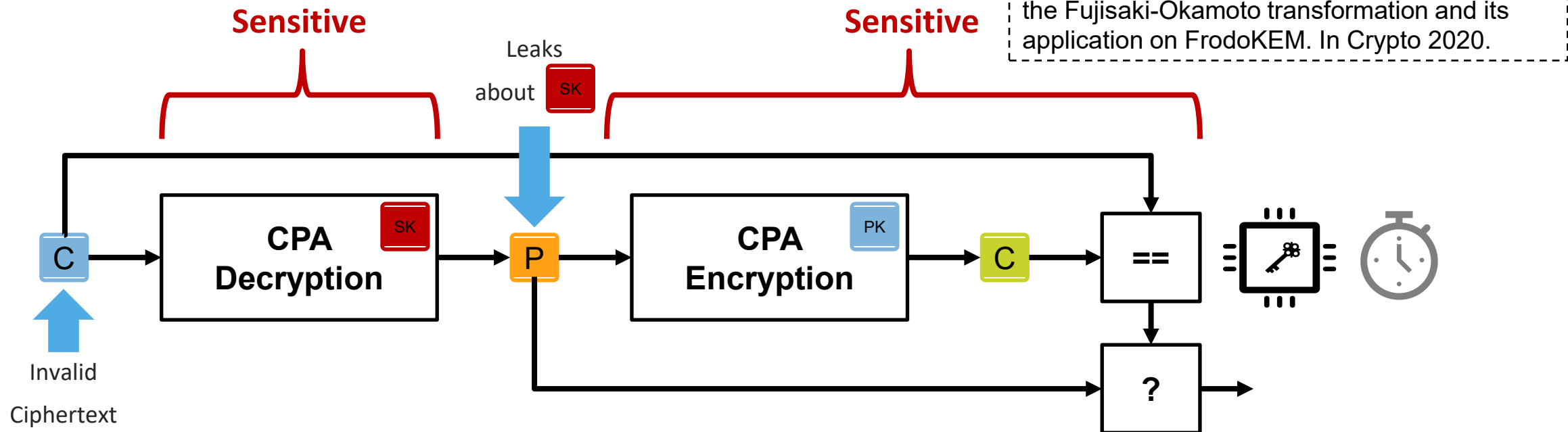
- Attacker inputs only valid ciphertexts
- Attack focuses on **CPA Decryption**, everything after (and including) **P** is public
- Only need to protect **CPA Decryption**



## THE SCA PROBLEM OF THE FO-TRANSFORM

### Attack 2: Chosen Ciphertext

- Attacker inputs specially-crafted invalid ciphertexts
- Attack focuses on **CPA Decryption** + everything after (and including) **P** is potentially sensitive
- Potentially all (or most) modules need to be hardened





## THE SCA PROBLEM OF THE FO-TRANSFORM



Why is it bad?



Millions of Points of Interest (Pol)



Low number of leakage classes (worst case = 2)



Easy to build templates

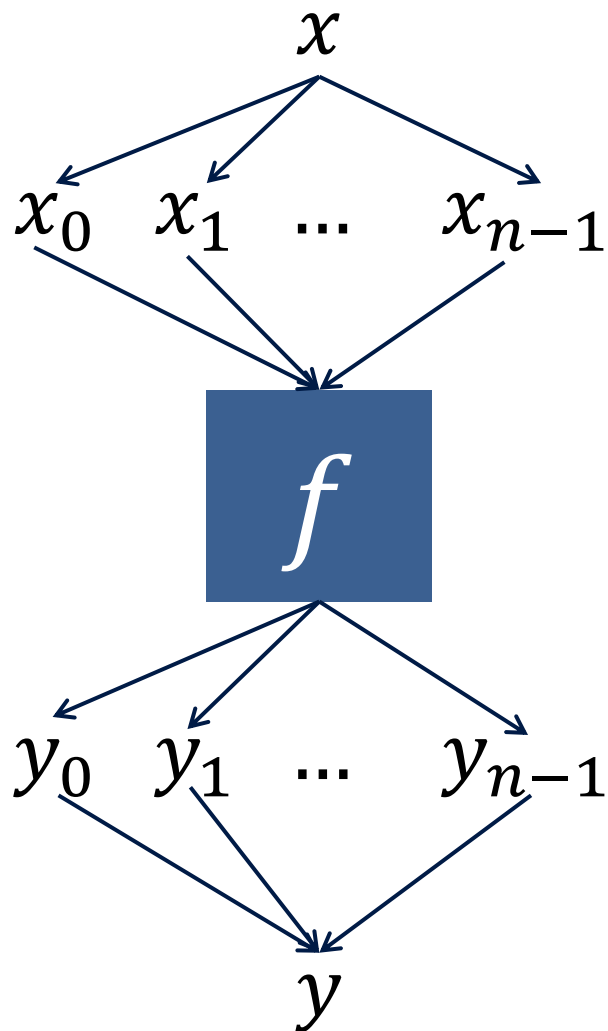
# SIDE-CHANNEL ATTACKS ON THE FO-TRANSFORM



- Ravi et al. “Generic Side-channel attacks on CCA-secure lattice-based PKE and KEMs” TCHES 2020
- Xu et al. “Magnifying Side-Channel Leakage of Lattice-Based Cryptosystems with Chosen Ciphertexts: The Case Study of Kyber” IEEE Transactions on Computers, 2021
- Qin et al. “A Systematic Approach and Analysis of Key Mismatch Attacks on Lattice-Based NIST Candidate KEMs” ASIACRYPT 2021
- Ngo et al. “A Side-Channel Attack on a Masked IND-CCA Secure Saber KEM Implementation” TCHES 2021
- Ravi et al. “Will You Cross the Threshold for Me? - Generic Side-Channel Assisted Chosen-Ciphertext Attacks on NTRU-based KEMs” TCHES 2022
- Ueno et al. “Curse of Re-encryption: A Generic Power/EM Analysis on Post-Quantum KEMs” TCHES 2022
- Shen et al. “Find the Bad Apples: An efficient method for perfect key recovery under imperfect SCA oracles – A case study of Kyber” IACR ePrint archive 2022
- Ngo et al. “Side-Channel Attacks on Lattice-Based KEMs Are Not Prevented by Higher-Order Masking” IACR ePrint archive 2022
- Rajedran et al. “Pushing the Limits of Generic Side-Channel Attacks on LWE-based KEMs - Parallel PC Oracle Attacks on Kyber KEM and Beyond” IACR ePrint archive 2022
- ...



# MASKING AGAINST SIDE-CHANNEL ATTACKS



- Encode sensitive variables into shares
- Compute securely on shares
- Decode at end to recover result

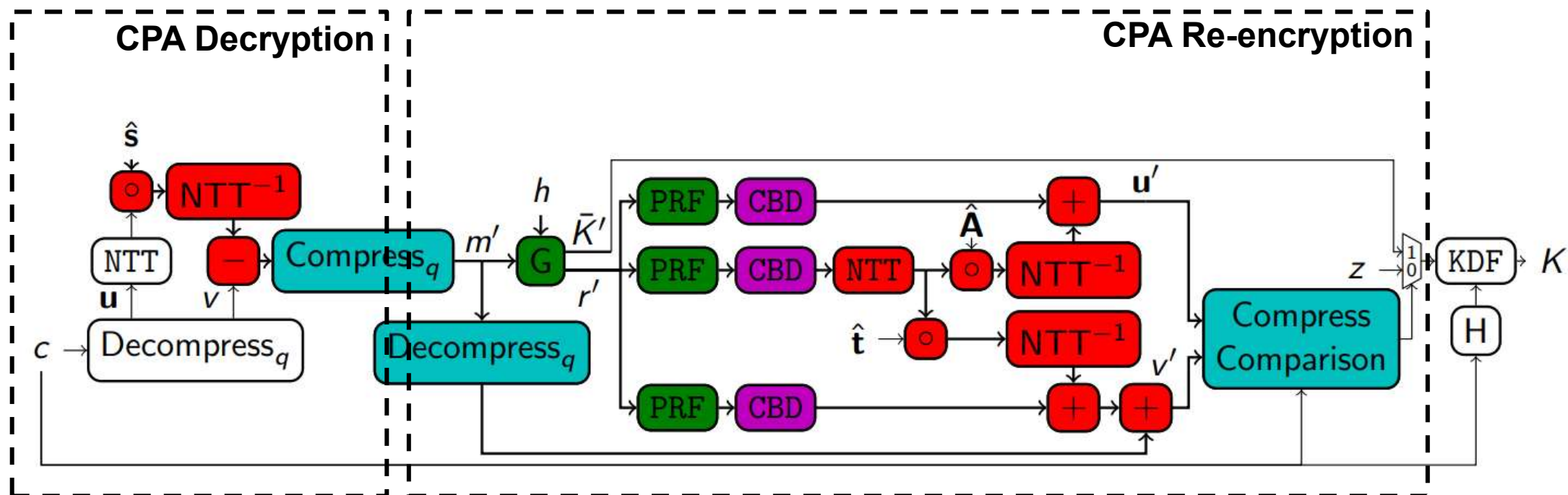
Masking if implemented **correctly** increases the attack complexity **exponentially** in the number of shares.

(assuming sufficient noise)

$$x = x_0 + x_1 \bmod q \quad (\textit{arithmetic masking})$$

$$x = x_0 \oplus x_1 \oplus x_2 \oplus x_3 \quad (\textit{Boolean masking})$$

# MASKING KYBER



Poly. arithmetic (■):

- Arith. masking.
- Linear overheads.

Hash functions (■):

- Boolean masking.
- Quadratic overheads.

Poly. sampl. (■) & compress. (■):

- Boolean & arith. masking.
- Quadratic overheads.

# MASKING KYBER

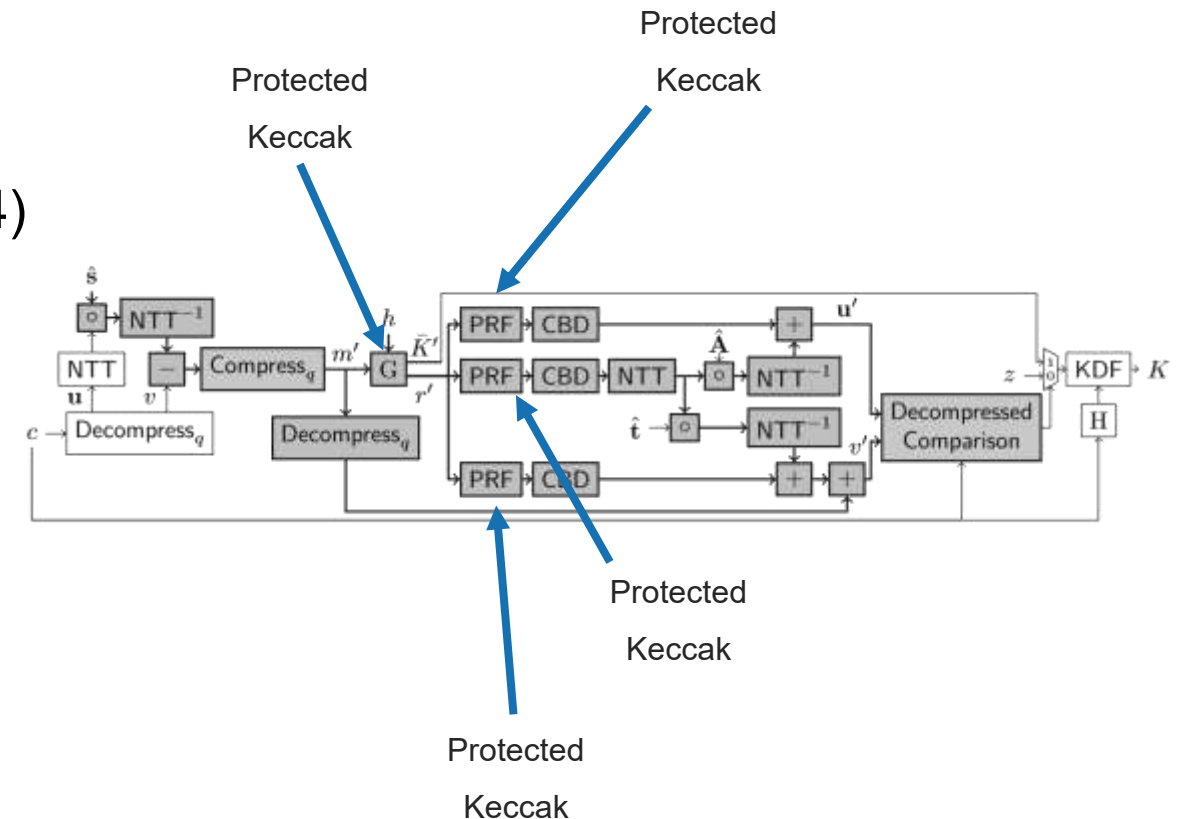


What is the bottleneck for masking Kyber?

## Latest Performance Numbers from [BG22]:

- Bitsliced masked Kyber (pure SW, ARM Cortex-M4)
- Performance values for 3 shares:

Masked Decapsulation	16.7 M Cycles (100%)
Keccak	7.22 M Cycles (43%)
B2A Conversion	5.02 M Cycles (30%)
Rest	4.46 M Cycles (27%)



# MASKING KYBER

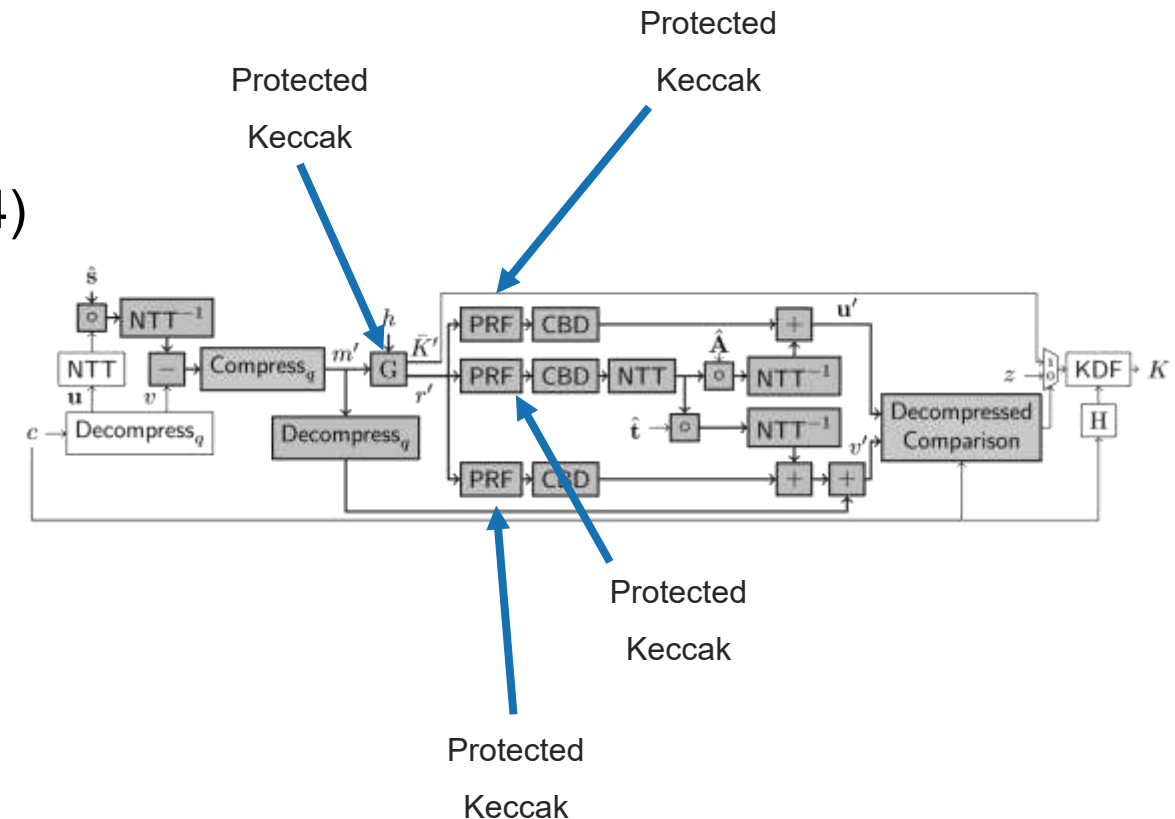


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**Most of the protected Keccak calls are in the re-encryption.**



## A CLOSER LOOK AT THE MASKED DECAPSULATION

Table 4: STM32F4 ARM Cortex-M4 MCU Performance numbers for masked Kyber.CCAKEM.Dec and its subroutines in kCycles.

Operation	Number of shares					
	2	3	4	5	6	7
Kyber.CCAKEM.Decaps	3 178	57 141	97 294	174 220	258 437	350 529
Kyber.CPAPKE.Dec	200	4 203	7 047	13 542	20 323	27 230
Kyber.CPAPKE.Enc	2 024	18 879	32 594	53 298	75 692	104 191
comparison ( $c = c'$ )	693	32 293	54 725	102 922	156 075	210 518
$\mathcal{G}$	98	1 639	2 801	4 489	6 456	8 794
$\mathcal{H}$	113	113	113	113	113	113
$\mathcal{H}'$	13	13	13	13	13	13



- Masked decryption is <10% of the cost of masked decapsulation
- Cost of masked decapsulation is dominated by the masked FO

## A VERY SIMPLE IDEA



Replace expensive FO by a signature verification of the ciphertext.

Signature verification only uses public data and does not require SCA protection.



Never decrypt untrusted ciphertexts.

- Based on the *Encrypt-then-Sign* ( $\mathcal{EtS}$ ) paradigm
- CCA security shown in [ADR02] in the outsider security model
- Post-quantum CCA security shown in [CPPS20]

- Y. Zheng. *Signcryption and its applications in efficient public key solutions*. ISW 1997.
- Azouaoui, M., Kuzovkova, Y., Schneider, T., van Vredendaal, C. *Post-Quantum Authenticated Encryption against Chosen-Ciphertext Side-Channel Attacks*. TCHES 2022.
- An, JH., Dodis, Y., Rabin, R. *On the Security of Joint Signature and Encryption*. EUROCRYPT 2002.
- Chatterjee, S., Pandit, T., Puria, SKP., Shah, A. *Signcryption in a Quantum World*. IACR ePrint Arch., 2020.

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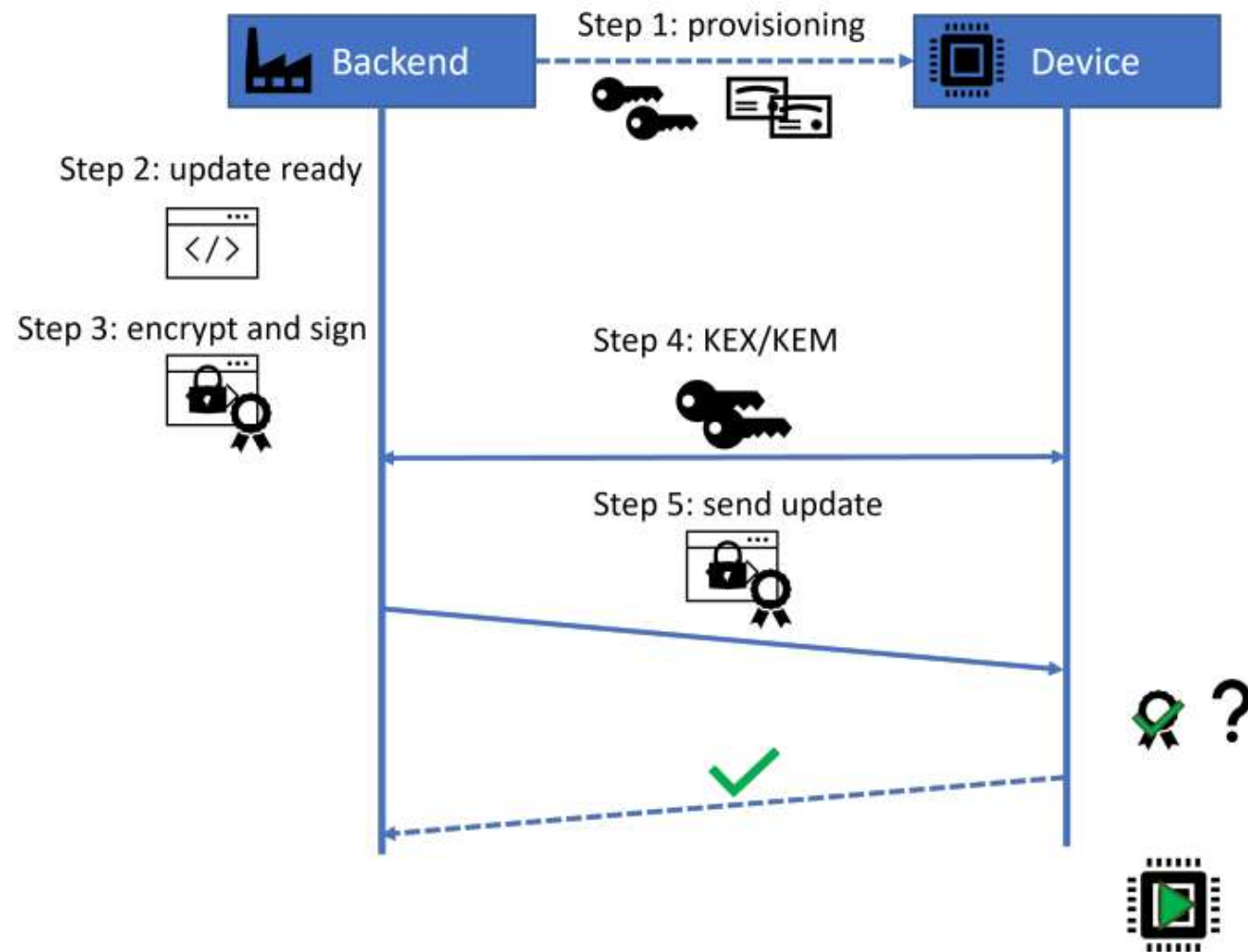
Never decrypt untrusted ciphertexts.

Adversary has only access to public material.  
It is neither the sender nor the receiver.

- Based on the *Encrypt-then-Sign* ( $\mathcal{EtS}$ ) paradigm
- CCA security shown in [ADR02] in the outsider security model
- Post-quantum CCA security shown in [CPPS20]

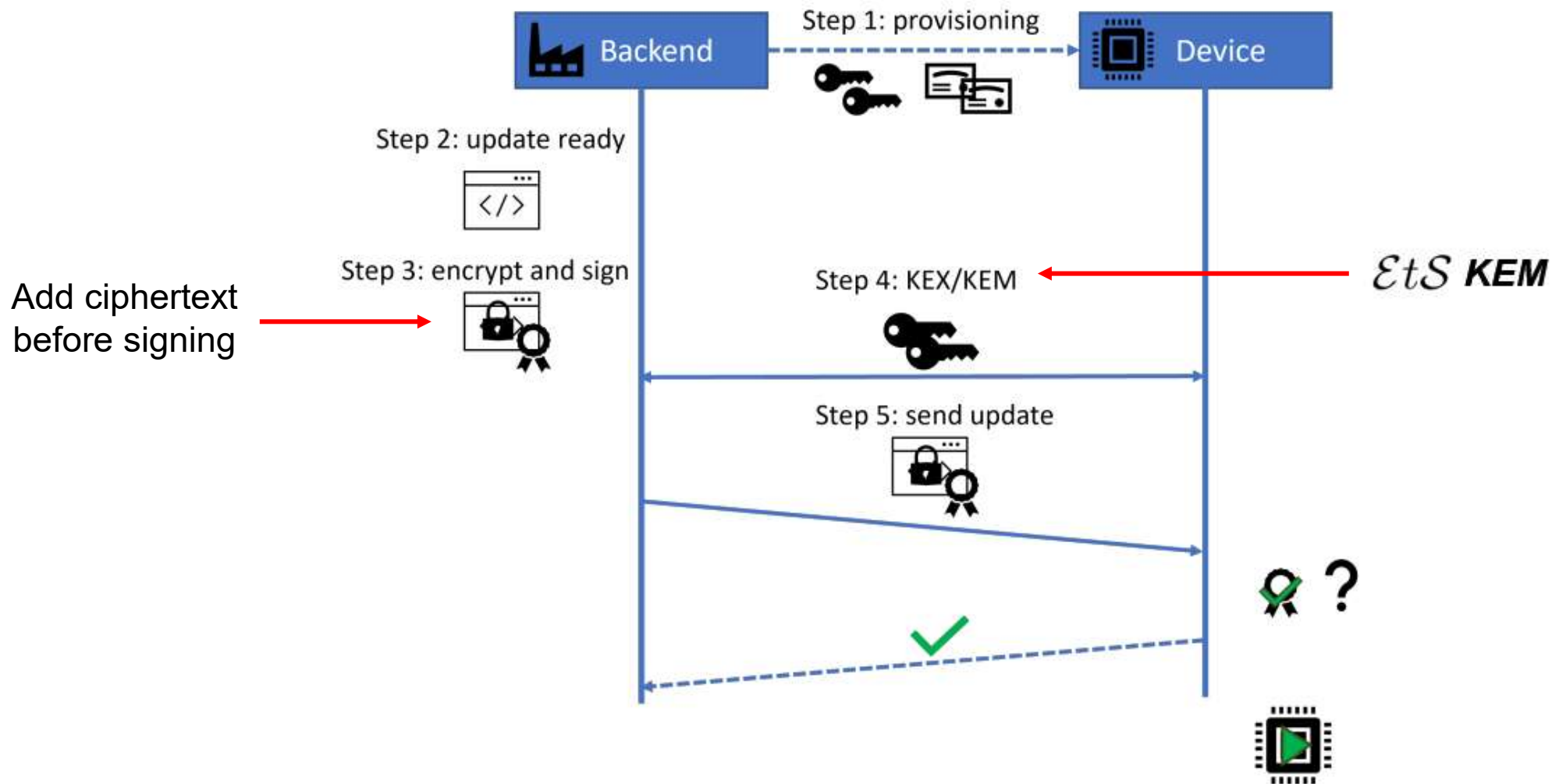
- Y. Zheng. *Signcryption and its applications in efficient public key solutions*. ISW 1997.
- Azouaoui, M., Kuzovkova, Y., Schneider, T., van Vredendaal, C. *Post-Quantum Authenticated Encryption against Chosen-Ciphertext Side-Channel Attacks*. TCHES 2022.
- An, JH., Dodis, Y., Rabin, R. *On the Security of Joint Signature and Encryption*. EUROCRYPT 2002.
- Chatterjee, S., Pandit, T., Puria, SKP., Shah, A. *Signcryption in a Quantum World*. IACR ePrint Arch., 2020.

# THE *EtS* KEM FOR SECURE UPDATE MECHANISM

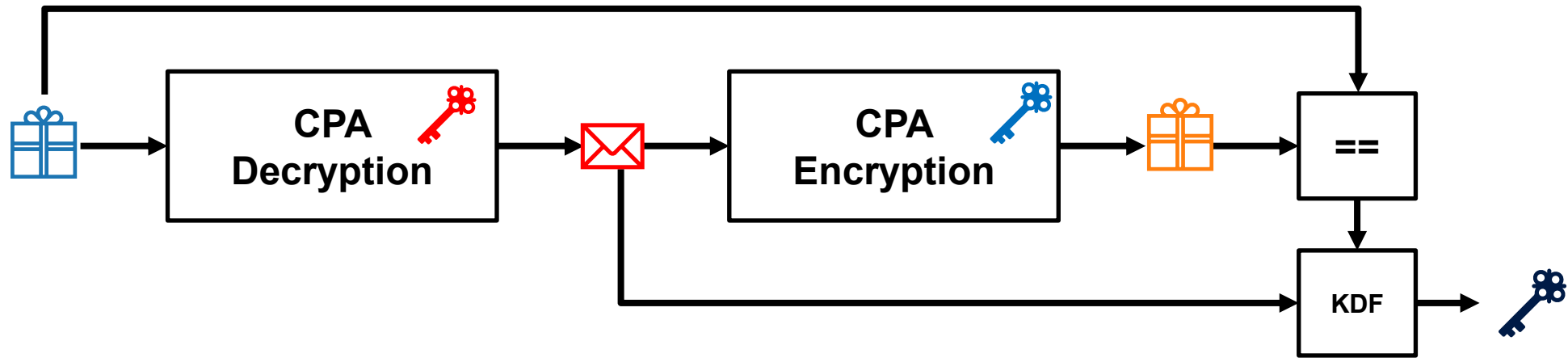




# THE $\mathcal{EtS}$ KEM FOR SECURE UPDATE MECHANISM

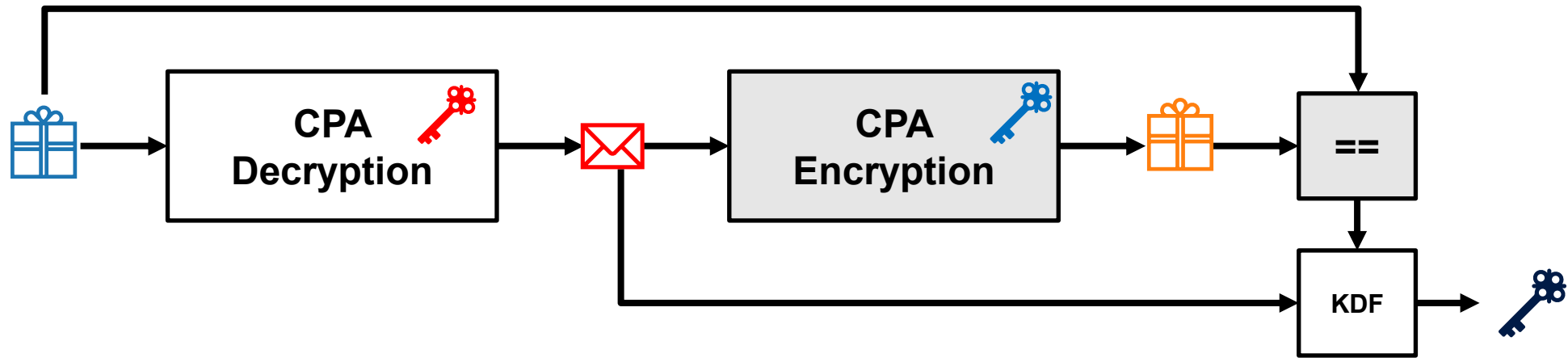


## THE $\mathcal{EtS}$ KEM VS. THE FO KEM



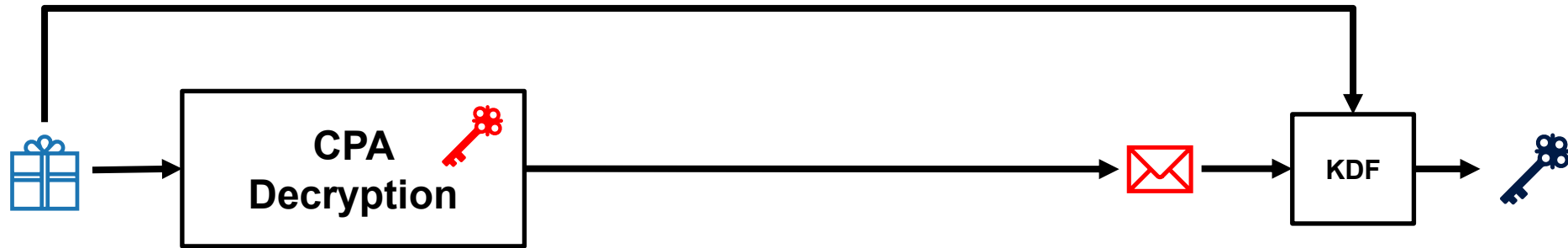
- CCA FO KEM Decapsulation -

## THE $\mathcal{EtS}$ KEM VS. THE FO KEM



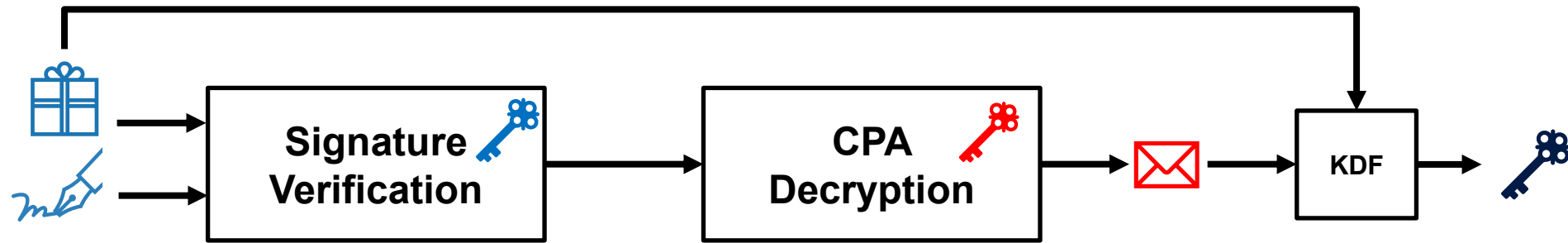
- CCA FO KEM Decapsulation -

## THE $\mathcal{EtS}$ KEM VS. THE FO KEM



- CPA PKE Decryption -

## THE $\mathcal{EtS}$ KEM VS. THE FO KEM

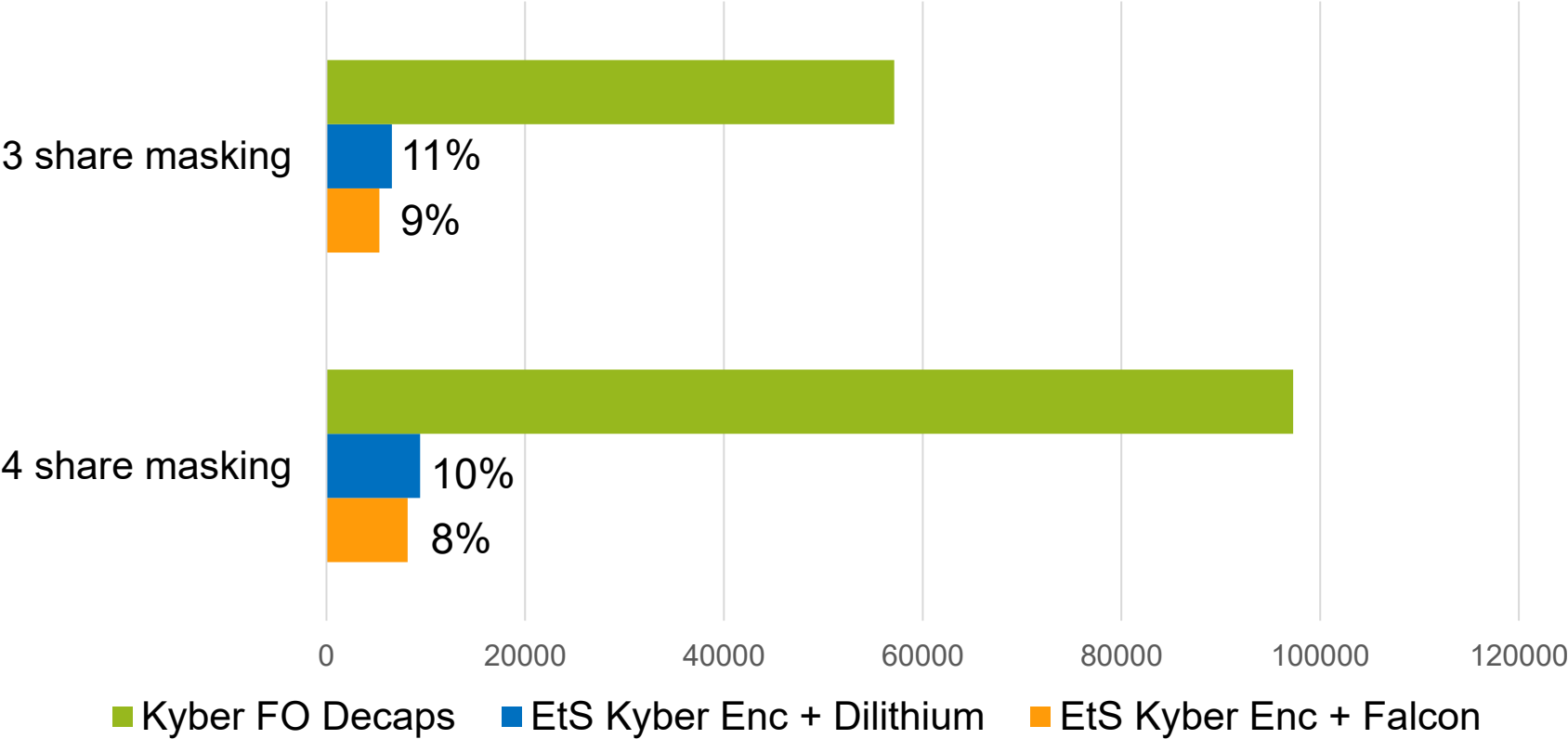


- CCA  $\mathcal{EtS}$  KEM Decapsulation -



# THE *EtS* KEM VS. THE FO KEM

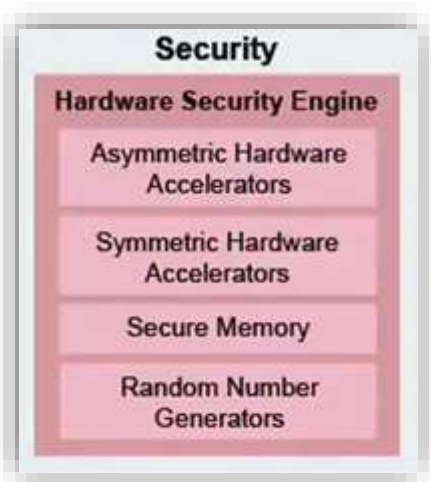
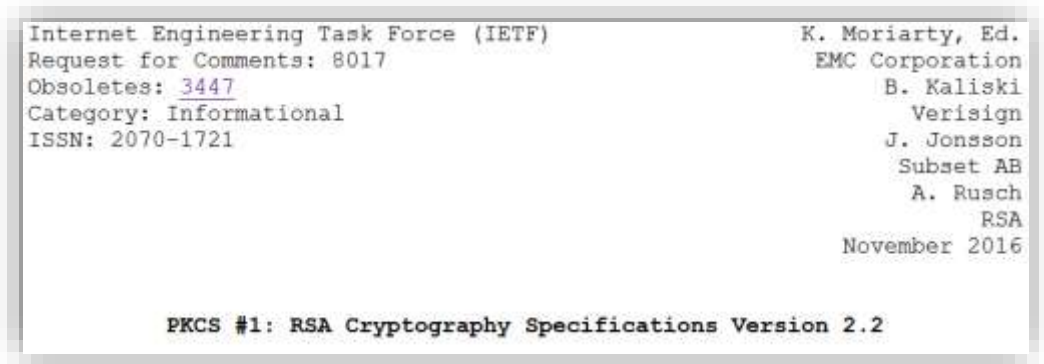
EtS KEM vs FO KEM in kCycles



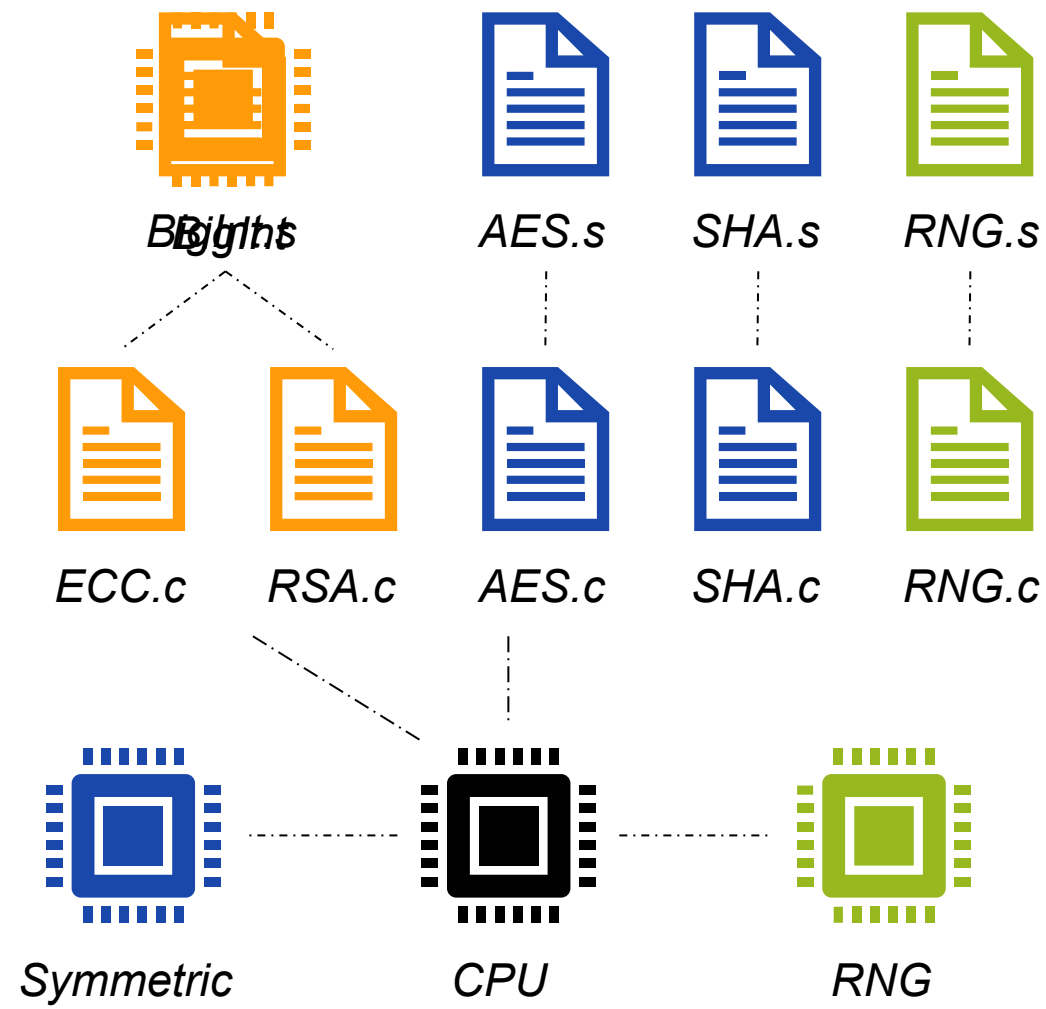
Ciphertext size	1088 bytes	4381 bytes	2368 bytes
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# PQC & HW RE-USE

# IMPLEMENTING CLASSICAL CRYPTOGRAPHY



*S32G2 automotive processor spec*



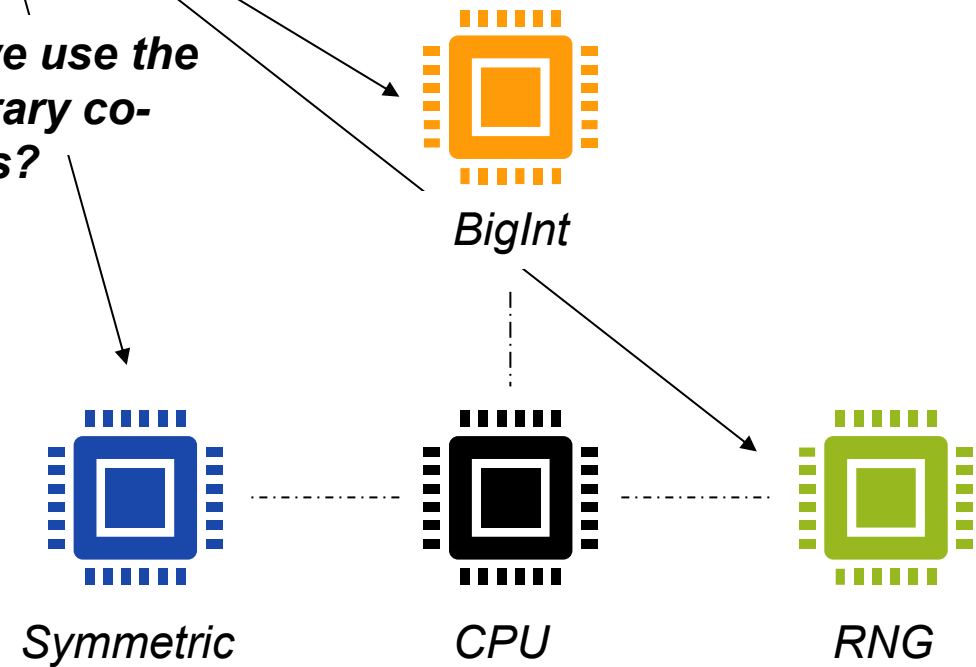
1. <https://www.nxp.com/products/processors-and-microcontrollers/arm-processors/s32g-vehicle-network-processors:S32G-PROCESSORS>

# IMPLEMENTING POST-QUANTUM CRYPTOGRAPHY



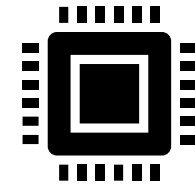
Lattice-based winners: **Kyber, Dilithium, Falcon** (Saber, NTRU, FrodoKEM)

*How can we use the contemporary co-processors?*

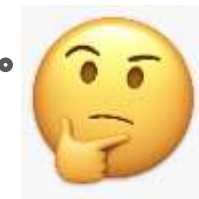


## RE-USING EXISTING HW

Approach	Core	Structure	Size
RSA	Modular multiplication	$(\mathbb{Z}/n\mathbb{Z})^*$	$n$ is 3072-bit
ECC	Elliptic curve scalar multiplication	$E(\mathbb{F}_p)$	$p$ is 256-bit
Lattice	Polynomial multiplication	$(\mathbb{Z}/q\mathbb{Z})[X]/(X^n + 1)$	$q$ is 16-bit $n$ is 256



Co-pro present in chips



## KRONECKER SUBSTITUTION

*Polynomial domain*

$$f = 1 + 2x + 3x^2 + 4x^3$$

$$g = 5 + 6x + 7x^2 + 8x^3$$

×

$$fg = \underline{5} + \underline{16x} + \underline{34x^2} + \underline{60x^3} + \underline{61x^4} + \underline{52x^5} + \underline{32x^6}$$

*Kronecker domain (with evaluation point 100)*

$$f(100) = 4030201$$

$$g(100) = 8070605$$

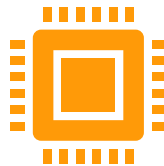
×

$$fg(100) = \underline{32526160341605}$$

Grundzüge einer arithmetischen Theorie der  
algebraischen Grössen.

(Von L. Kronecker.)

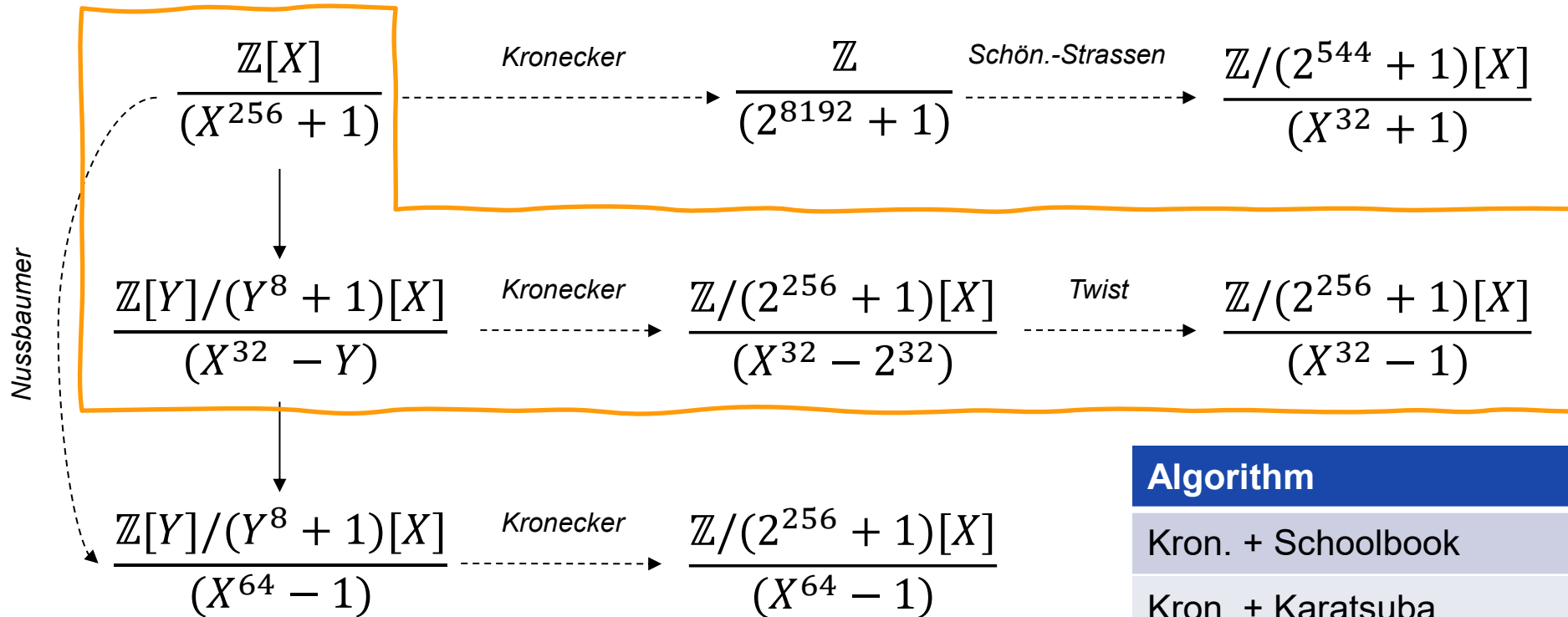
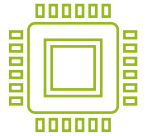
(Abdruck einer Festschrift zu Herrn E. E. Kummers Doctor-Jubiläum, 10. September 1881.)





# POLYNOMIAL MULTIPLICATION TECHNIQUES

Kronecker evaluation at  $2^{32}$   
 Multiplication with a **256-bit** multiplier



**Kronecker+**

Algorithm	# Muls	# Bits
Kron. + Schoolbook	1024	256
Kron. + Karatsuba	243	256
Kron. + Toom-Cook	63	256
Kron. + Schön.-Strassen	32	544
Nussbaumer + Kron.	64	256
<b>Kronecker+</b>	<b>32</b>	<b>256</b>

- Harvey. Faster polynomial multiplication via multipoint Kronecker substitution. J. of Sym. Comp. 2009.
- Albrecht, Hanser, Hoeller, Pöppelmann, Virdia, Wallner; Implementing RLWE-based schemes using an RSA co-processor. TCHES 2019
- Bos, Renes, van Vredendaal: Polynomial Multiplication with Contemporary Co-Processors: Beyond Kronecker, Schönhage-Strassen & Nussbaumer. USENIX 2022.

## CAN WE USE EXISTING HARDWARE

- Works very well for Saber, ~8-10x faster for matrix / vector multiplication on RISC-V

Function	Ref.	$\tau$				
		0	1	2	3	4
MatrixVectorMul	2 468	716	430	295	255	291
InnerProd	823	235	138	91	76	84
indcpa_kem_keypair	3 691	1 972	1 682	1 549	1 509	1 548
indcpa_kem_enc	4 477	2 152	1 765	1 585	1 528	1 574
indcpa_kem_dec	856	286	189	144	129	138
crypto_kem_keypair	4 018	2 300	2 011	1 877	1 837	1 876
crypto_kem_enc	5 280	2 958	2 571	2 391	2 334	2 380
crypto_kem_dec	5 786	2 893	2 411	2 184	2 113	2 168

*Cycle counts on RV32IMC in 1000s of cycles, rounded up*

- CRYSTALS Design: Sample matrix elements directly in NTT domain

# LOW-MEMORY PQC

## SECURE ELEMENTS AND END-TO-END SERVICES

NXP propels today's on-the-go lifestyle with intelligent mobile solutions that safely connect consumers and their technology to the world around them.



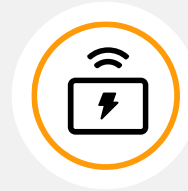
SECURE ELEMENTS  
AND END-TO-END  
SERVICES



CUSTOM HIGH-  
PERFORMANCE  
INTERFACES



SMART VOICE,  
AUDIO, AND HAPTIC  
SOLUTIONS



EFFICIENT  
CHARGING  
SOLUTIONS



### DEFINING WHAT'S NEXT FOR MOBILE PHONES

NXP has been driving the mobile wallet expansion, advancing analog and charging solutions add more capabilities to mobile phones, notebooks, and tablets.

- NFC, eSE, eSIM, and UWB solutions
- Advanced analog solutions for personal computing
- Fast charging with USB Type-C



### WEARABLES

Thanks to secure mobile payments, advanced audio solutions and tailored MCUs, wearables naturally blend into our lives.

- NFC+eSE mobile wallet solutions
- Highly integrated Arm® based MPUs and MCUs
- MiGLO™ NFMI radios for wireless audio



### ACCESSORIES

NXP's anti-counterfeiting technology, among others products, support charging cables, power adapters, and wireless charging pads for mobile phones to help OEMs protect their brand and provides safety to their customers by making trusted accessories.



## INDUSTRIAL



Fit-for-purpose Scalable Processors



Functional Safety & Security



Industrial Connectivity & Control



Machine Learning & Vision



Comprehensive Software

## PQC ON EMBEDDED DEVICES

What is embedded?

- NIST has recommended a focus on the Arm Cortex-M4

**Pqm4:** Post-quantum crypto library for the ARM Cortex-M4, STM32F4DISCOVERY

**196 KiB of RAM and 1 MiB of Flash ROM**

Low-power Edge computing: LPC800 Series

- 8 to 60 MHz Cortex-M0+ core
- { 4, 8, 16 } KiB of SRAM
- { 16, 32 } KiB Flash

The fastest implementations in pqm4 require  **$\approx 49$ ,  $\approx 80$  and  $\approx 116$  KiB memory** for Dilithium- $\{2,3,5\}$ .



## DILITHIUM SIGNATURE GENERATION

---

**Algorithm 2** Dilithium signature generation (taken from [18])

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Polynomials from

$$R_q = \mathbb{Z}_q[X]/(X^{256} + 1)$$

where  $q = 2^{23} - 2^{13} + 1$  and stored as 32-bit values.

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**Dilithium-3:**  $(k, \ell) = (6, 5)$

(Re-)generate matrix  $\mathbf{A}$  and  $\mathbf{y}$  on-the-fly

- Reduce by  $k \cdot \ell$  KB for  $\mathbf{A}$   
→ **30 KB**
- Reduce by  $\ell$  KB for  $\mathbf{y}$   
→ **5 KB**

## DILITHIUM SIGNATURE GENERATION

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**Dilithium-3:**  $(k, \ell) = (6, 5)$

(Re-)generate matrix  $\mathbf{A}$  and  $\mathbf{y}$  on-the-fly: 80, 45 KB

Compress  $\mathbf{w}$

- Store values as 24-bit
- One  $R_q$  elements needs 768 bytes
- Packing and unpacking is simple and efficient
- Reduces memory by Reduce by 256k bytes → 1.5 KB



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8:    $\mathbf{w}_1 := \text{HighBits}_q(\mathbf{w}, 2\gamma_2)$ 
9:    $\tilde{c} \in \{0, 1\}^{256} := H(\mu \parallel \mathbf{w}_1)$ 
10:   $c \in B_\infty := \text{SampleInBall}(\tilde{c})$  ▷ Store  $c$  in NTT representation as  $\hat{c} = \text{NTT}(c)$ 
11:   $\mathbf{z} := \mathbf{y} + c\mathbf{s}_1$  ▷ Compute  $c\mathbf{s}_1$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{s}}_1)$ 
12:   $\mathbf{r}_0 := \text{LowBits}_q(\mathbf{w} - c\mathbf{s}_2, \gamma_2)$  ▷ Compute  $c\mathbf{s}_2$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{s}}_2)$ 
13:  if  $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$  or  $\|\mathbf{r}_0\|_\infty \geq \gamma_2 - \beta$  then
14:     $(\mathbf{z}, \mathbf{h}) := \perp$ 
15:  else
16:     $\mathbf{h} := \text{MakeHint}_q(-c\mathbf{t}_0, \mathbf{w} - c\mathbf{s}_2 + c\mathbf{t}_0, 2\gamma_2)$  ▷ Compute  $c\mathbf{t}_0$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{t}}_0)$ 
17:    if  $\|c\mathbf{t}_0\|_\infty \geq \gamma_2$  or the # of 1's in  $\mathbf{h}$  is greater than  $\omega$  then
18:       $(\mathbf{z}, \mathbf{h}) := \perp$ 
19:   $\kappa := \kappa + \ell$ 
20: return  $\sigma = (\tilde{c}, \mathbf{z}, \mathbf{h})$ 

```

Polynomials from

$$R_q = \mathbb{Z}_q[X]/(X^{256} + 1)$$

where  $q = 2^{23} - 2^{13} + 1$  and stored as 32-bit values.

→ One  $R_q$  elements needs **1KB**

**Dilithium-3:**  $(k, \ell) = (6, 5)$

(Re-)generate matrix  $\mathbf{A}$  and  $\mathbf{y}$  on-the-fly: ~~80 KB~~ → 45 KB

Compress  $\mathbf{w}$ : ~~45 KB~~ → 43.5 KB

Compressing multiplications

- NTT used for faster polynomial multiplication
- Secret key coefficient range is much smaller
- Not using NTT reduces by  $2k + \ell$  KB → **17 KB**

## DILITHIUM SIGNATURE GENERATION

### Algorithm 2 Dilithium signature generation (taken from [18])

**Input:** Secret key  $sk$  and a message  $M$ .

**Output:** Signature  $\sigma = \text{Sign}(sk, M)$ .

```

1:  $\mathbf{A} \in R_q^{k \times \ell} := \text{ExpandA}(\rho)$  ▷  $\mathbf{A}$  is generated in NTT domain as  $\hat{\mathbf{A}}$ 
2:  $\mu \in \{0, 1\}^{512} := H(tr \parallel M)$ 
3:  $\kappa := 0, (\mathbf{z}, \mathbf{h}) := \perp$ 
4:  $\rho' \in \{0, 1\}^{512} := H(K \parallel \mu)$  (or  $\rho' \leftarrow \{0, 1\}^{512}$  for randomized signing)
5: while  $(\mathbf{z}, \mathbf{h}) = \perp$  do ▷ Pre-compute  $\hat{\mathbf{s}}_1 := \text{NTT}(\mathbf{s}_1)$ ,  $\hat{\mathbf{s}}_2 := \text{NTT}(\mathbf{s}_2)$ , and  $\hat{\mathbf{t}}_0 := \text{NTT}(\mathbf{t}_0)$ 
6:    $\mathbf{y} \in S_{\gamma_1}^\ell := \text{ExpandMask}(\rho', \kappa)$ 
7:    $\mathbf{w} := \mathbf{A}\mathbf{y}$  ▷  $\mathbf{w} := \text{NTT}^{-1}(\hat{\mathbf{A}} \cdot \text{NTT}(\mathbf{y}))$ 
8:    $\mathbf{w}_1 := \text{HighBits}_q(\mathbf{w}, 2\gamma_2)$ 
9:    $\tilde{c} \in \{0, 1\}^{256} := H(\mu \parallel \mathbf{w}_1)$ 
10:   $c \in B_{\sigma} := \text{SampleInBall}(\tilde{c})$  ▷ Store  $c$  in NTT representation as  $\hat{c} = \text{NTT}(c)$ 
11:   $\mathbf{z} := \mathbf{y} + c\mathbf{s}_1$  ▷ Compute  $c\mathbf{s}_1$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{s}}_1)$ 
12:   $\mathbf{r}_0 := \text{LowBits}_q(\mathbf{w} - c\mathbf{s}_2, \gamma_2)$  ▷ Compute  $c\mathbf{s}_2$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{s}}_2)$ 
13:  if  $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$  or  $\|\mathbf{r}_0\|_\infty \geq \gamma_2 - \beta$  then
14:     $(\mathbf{z}, \mathbf{h}) := \perp$ 
15:  else
16:     $\mathbf{h} := \text{MakeHint}_q(-c\mathbf{t}_0, \mathbf{w} - c\mathbf{s}_2 + c\mathbf{t}_0, 2\gamma_2)$  ▷ Compute  $c\mathbf{t}_0$  as  $\text{NTT}^{-1}(\hat{c} \cdot \hat{\mathbf{t}}_0)$ 
17:    if  $\|c\mathbf{t}_0\|_\infty \geq \gamma_2$  or the # of 1's in  $\mathbf{h}$  is greater than  $\omega$  then
18:       $(\mathbf{z}, \mathbf{h}) := \perp$ 
19:   $\kappa := \kappa + \ell$ 
20: return  $\sigma = (\tilde{c}, \mathbf{z}, \mathbf{h})$ 

```

Polynomials from

$$R_q = \mathbb{Z}_q[X]/(X^{256} + 1)$$

where  $q = 2^{23} - 2^{13} + 1$  and stored as 32-bit values.

→ One  $R_q$  elements needs **1KB**

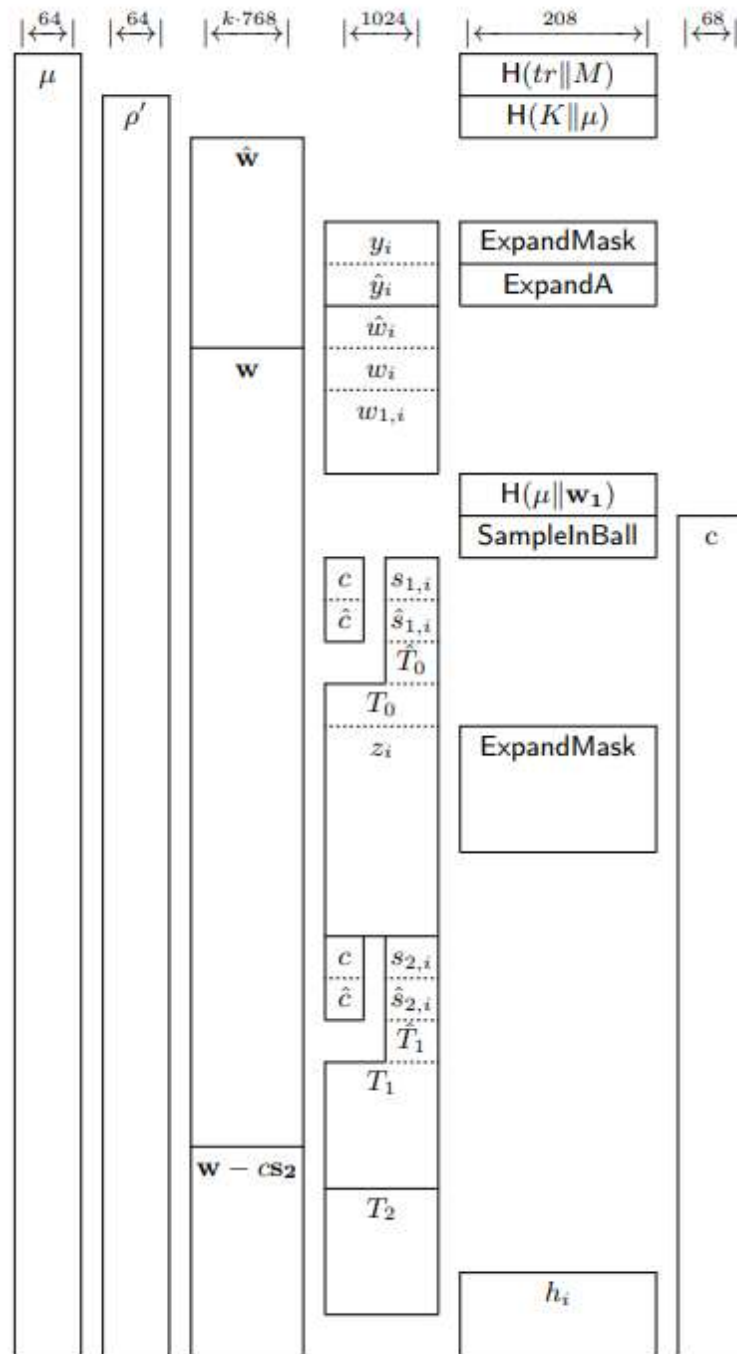
**Dilithium-3:**  $(k, \ell) = (6, 5)$

(Re-)generate matrix  $\mathbf{A}$  and  $\mathbf{y}$  on-the-fly: ~~80 KB~~ → 45 KB

Compress  $\mathbf{w}$ : ~~45 KB~~ → 43.5 KB

Compressing multiplications ~~43.5 KB~~ → 26.5 KB

Variable Allocation



```

 $\mu := H(tr \| M)$ 
 $\rho' := H(K \| \mu)$ 
 $\hat{w} := 0$ 

reject:
 $0 \leq i < \ell$ 
 $y_i := \text{ExpandMask}(\rho', \kappa)$ 
 $\hat{y}_i := \text{NTT}(y_i)$ 
 $\hat{w}_j := \hat{w}_j + \hat{A}_{j,i} \circ \hat{y}_i$  for  $0 \leq j < k$ 
 $0 \leq i < \ell$ 
 $w_i := \text{NTT}^{-1}(\hat{w}_i)$ 
 $w_{1,i} := \text{Highbits}(w_i)$ 
   $\triangleright$  store packed  $w_1$  in output buffer
 $\tilde{c} := H(\mu \| w_1)$   $\triangleright$  write  $\tilde{c}$  to signature
 $c := \text{SampleInBall}(\tilde{c})$ 
   $\triangleright$  make 16-bit  $c$  and  $s_{1,i}$  polynomials
 $\hat{c} := \text{NTT}_{q'}(c)$ ;  $\hat{s}_{1,i} = \text{NTT}_{q'}(s_{1,i})$ 
 $\hat{T}_0 := \hat{c} \circ \hat{s}_{1,i}$ 
 $0 \leq i < k$ 
 $T_0 := \text{NTT}_{q'}^{-1}(\hat{T}_0)$ 
   $\triangleright$  sample (using  $\text{ExpandMask}$ ) and
  and add  $y_i$  on-the-fly
 $z_i := T_0 + y_i$ 
  check  $\|z_i\|_\infty < \gamma_1 - \beta$ 
  write  $z_i$  to signature
 $0 \leq i < k$ 
   $\triangleright$  make 16-bit  $c$  and  $s_{2,i}$  polynomials
 $\hat{c} := \text{NTT}_{q'}(c)$ ;  $\hat{s}_{2,i} = \text{NTT}_{q'}(s_{2,i})$ 
 $\hat{T}_1 := \hat{c} \circ \hat{s}_{2,i}$ 
 $T_1 := \text{NTT}_{q'}^{-1}(\hat{T}_1)$ 
  check  $\|\text{LowBits}_q(w_i - T_1, 2\gamma_2)\|_\infty < \gamma_2 - \beta$ 
 $w_i - cs_{2,i} := w_i - T_1$ 
 $0 \leq i < k$ 
 $T_2 := c \cdot t_{0,i}$   $\triangleright$  schoolbook multiplication
  check  $\|T_2\|_\infty < \gamma_2$ 
 $h_i := \text{MakeHint}(-T_2, w_i - cs_{2,i} + T_2, 2\gamma_2)$ 
  write  $h_i$  to output

```

(Re-)generate matrix A and y on-the-fly: ~~80 KB~~  $\rightarrow$  45 KB

Compress w: ~~45 KB~~  $\rightarrow$  43.5 KB

Compressing multiplications ~~43.5 KB~~  $\rightarrow$  26.5 KB

Variable Allocation:

Total of

$64 + 64 + 768k + 1024 + 208 + 68$  bytes  $\rightarrow$  5268 bytes

In practice: 6.5 KB needed



DILITHIUM SIGNATURE GENERATION: LOW-MEMORY VERSION

Variant		Dilithium-3			
			KiB	Cc	
With asm	[7]	K	59.6	2,835	
		S	72.3	6,742	
		V	56.6	2,700	
	[1]	K	59.6	2,830	
		S	67.4	6,624	
		V	56.6	2,692	
C only	PQClean	K	59.4	3,504	
		S	77.7	12,987	
		V	56.4	3,666	
	New	K	6.4	5,112	1.8x
		S	6.5	36,303	5.5x
		V	2.7	7,249	2.7x

Smaller

Slower

# PQC Use-Cases





## AUTOMOTIVE



70%



70% connected cars by 2025

## INDUSTRIAL & IOT



12B



IoT Edge & end nodes from 6B units in '21 to 12B units in '25

## MOBILE



60B

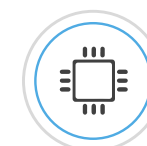


Tagging 60B products per year by 2025

## COMMUNICATION INFRASTRUCTURE

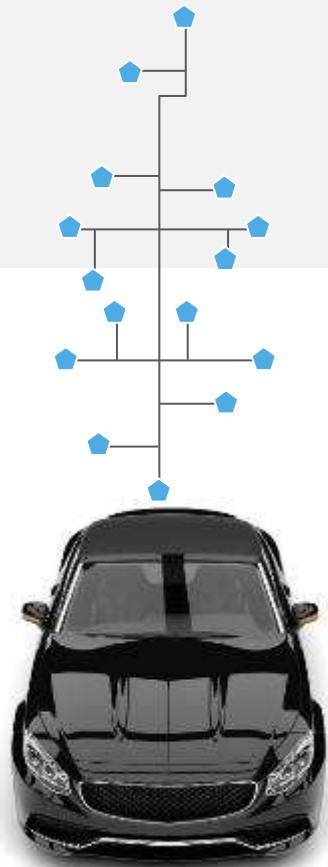


40B



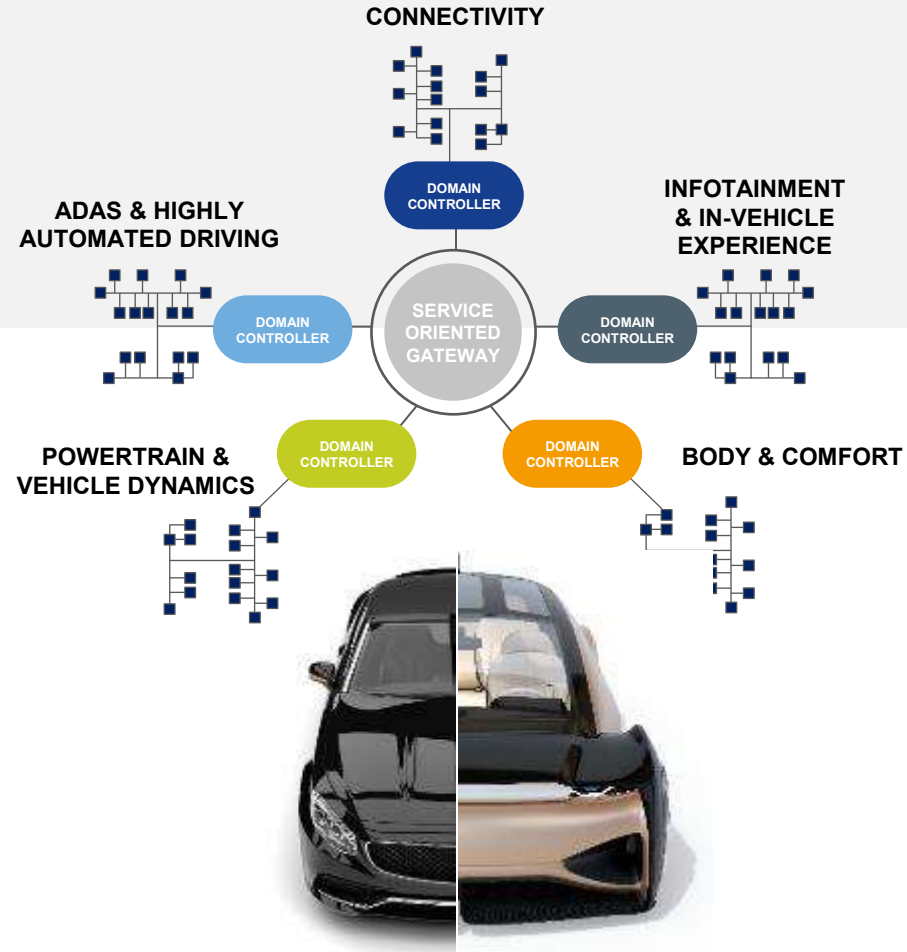
Secure anchors & services for 40B processors

# VEHICLE ARCHITECTURE TRANSFORMATION



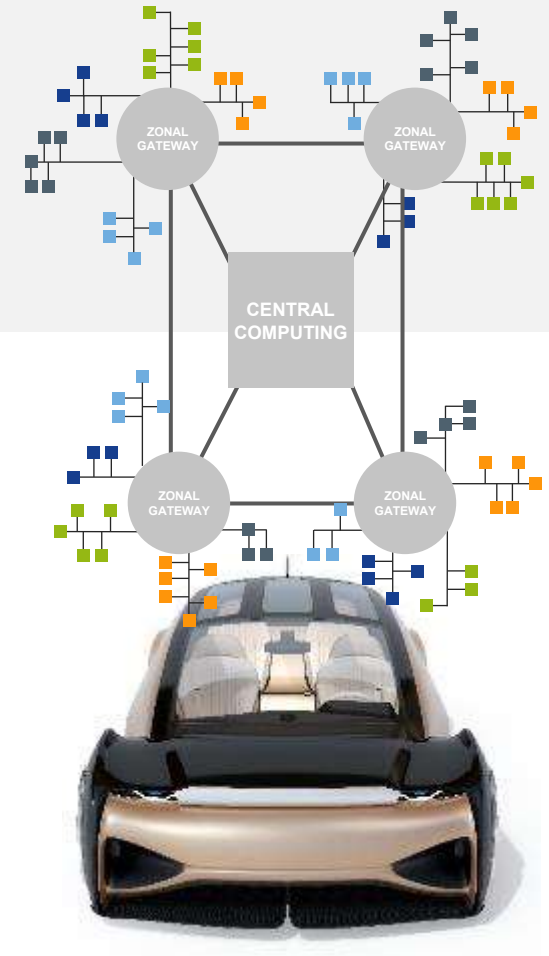
**TODAY | FLAT**

UNFIT FOR FUTURE MOBILITY



**LOGICAL RESTRUCTURE | DOMAINS**

ENABLING AUTONOMOUS CAR



**PHYSICAL RESTRUCTURE | ZONES**

ENABLING USER-DEFINED CAR

# NXP S32G2 VEHICLE NETWORK PROCESSOR WITH PQC INTEGRATION

## OUR TARGET PLATFORM: **S32G274A**

3 Lockstep Arm® Cortex®-M7  
Microcontrollers

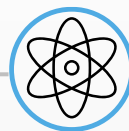
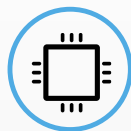
4 Cluster Lockstep Cortex-A53  
Microprocessors

8 MB of System RAM

Network Accelerators (LLCE/PFE)

Hardware Security Engine (HSE)

ASIL D Functional Safety Support



## POST-QUANTUM CRYPTO

Integrate PQC secure signature verification

Enable PQC secure boot

Secure Over-the-Air (OTA) updates

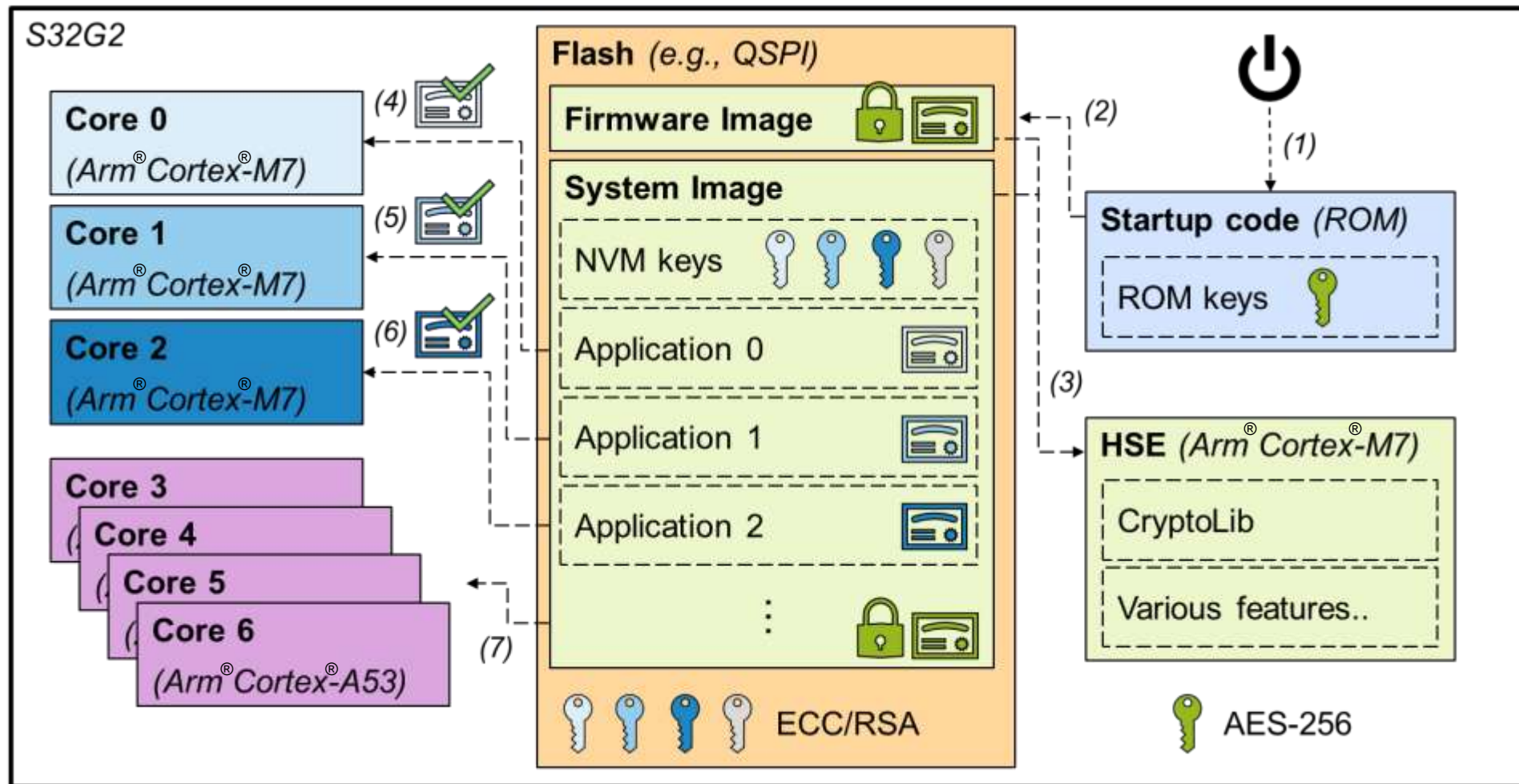
Secure vehicle and driver data



[www.nxp.com/S32G2](http://www.nxp.com/S32G2)

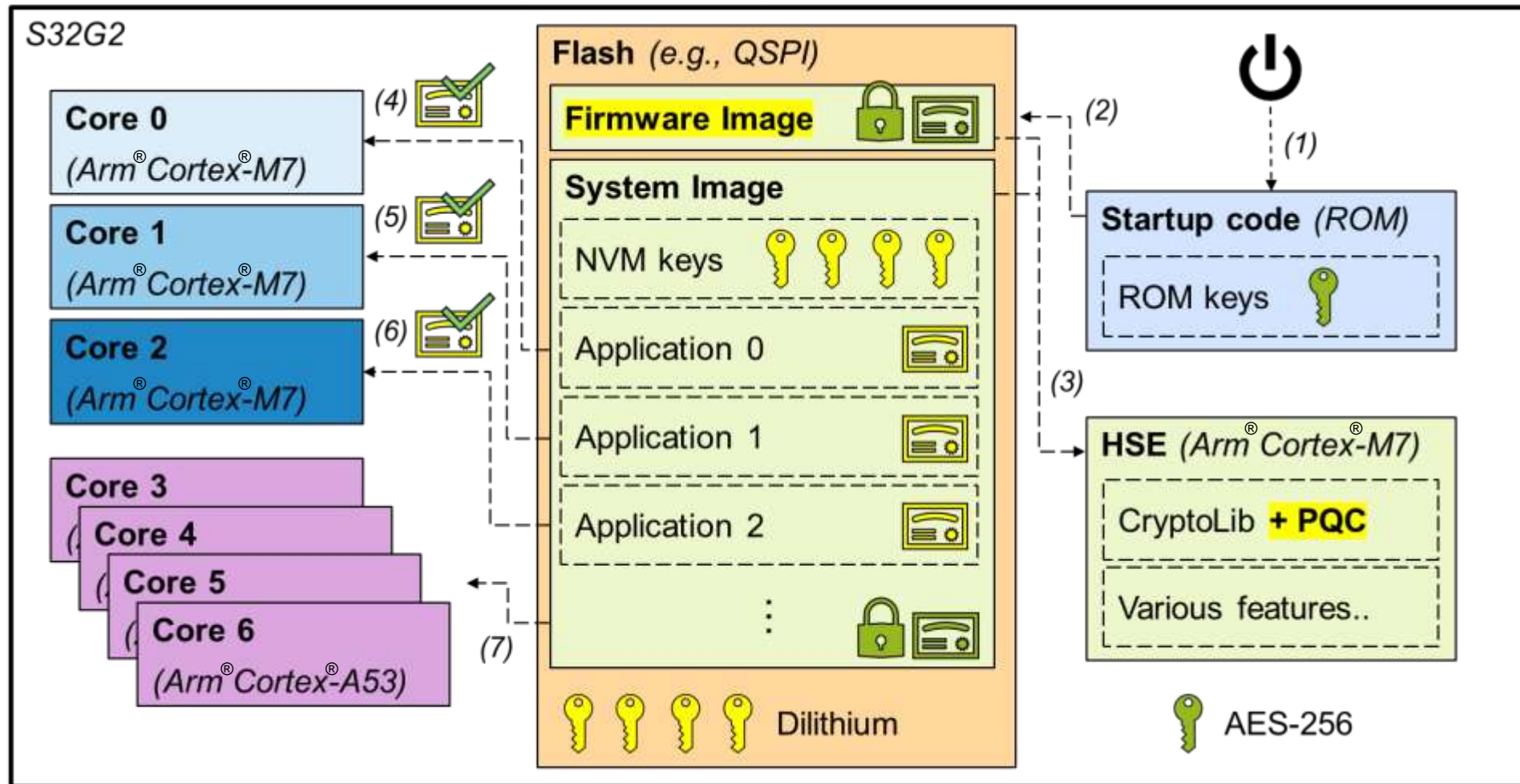


## PQC DEMO: HSE SECURE BOOT OVERVIEW

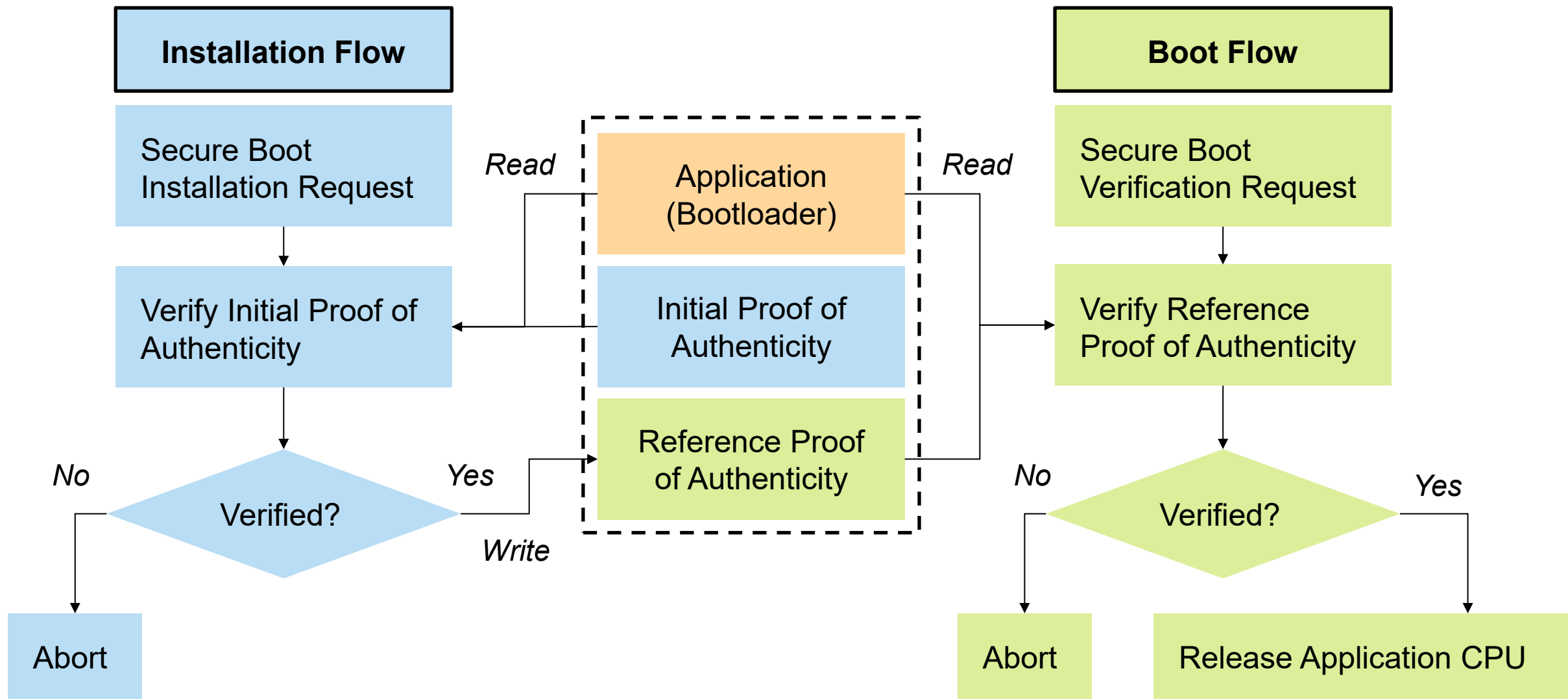




## PQC DEMO: HSE SECURE BOOT OVERVIEW



## S32G2 INSTALL VS BOOT (CONFIGURABLE)





## BENCHMARKS FOR AUTHENTICATION OF FW SIGNATURE ON THE S32G2

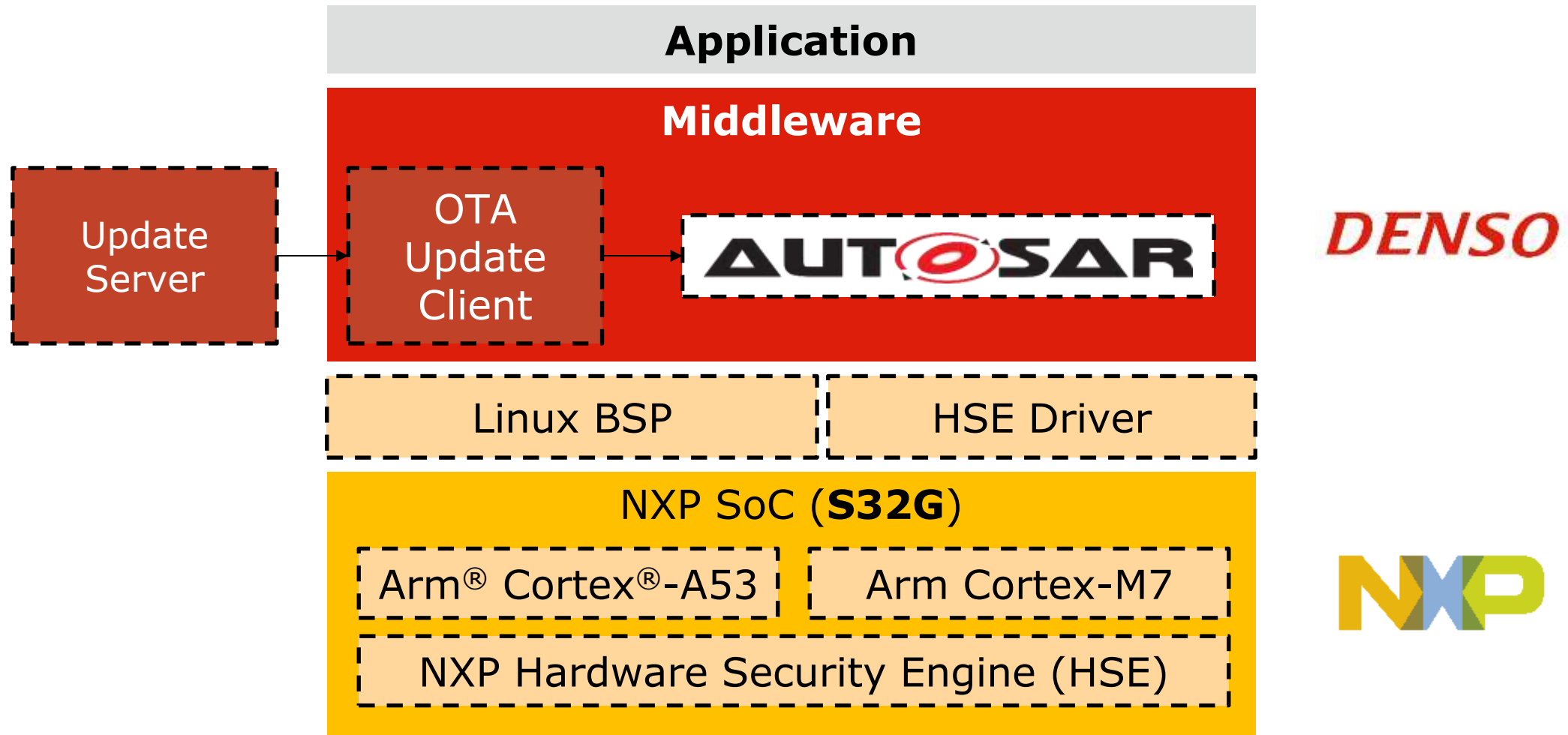
Alg.	Size		Performance (ms)			
			1 KB		128 KB	
	PK	Sig.	Inst.	Boot	Inst.	Boot
RSA 4K	512	512	2.6	0.0	2.7	0.2
ECDSA-p256	64	64	6.2	0.0	6.4	0.2
<b>Dilithium-3</b>	<b>1952</b>	<b>3293</b>	<b>16.7</b>	<b>0.0</b>	<b>16.9</b>	<b>0.2</b>



- Demonstrator only, further optimizations are possible (such as hardware accelerated SHA-3)
- Signature verification only required once for installation!
- During boot the signature verification can be replaced with a check of the Reference Proof of Authenticity

Bos, Carlson, Renes, Rotaru, Sprenkels, Waters: Post-Quantum Secure Boot on Vehicle Network Processors. Embedded Security in Cars. Escar 2022

# NXP + DENSO : PQC SECURE OVER-THE-AIR (OTA) UPDATE





## CONCLUSIONS

- Migration to PQC is a difficult & hot topic
- Many practical challenges
  - Memory
  - Available hardware (co-processors)
  - Efficient side-channel countermeasures

For automotive

- ✓ Large key sizes no issue, marginal increase in stack usage
- SHA-3 performance crucial, hardware acceleration important
- Little impact on OTA time (verification time not critical)
- Transition to PQC practical



# THANK YOU.

QUESTIONS?



SECURE CONNECTIONS  
FOR A SMARTER WORLD





SECURE CONNECTIONS  
FOR A SMARTER WORLD