High-Performance Scalar Multiplication using 8-Dimensional GLV/GLS Decomposition

Joppe W. Bos, Craig Costello, Huseyin Hisil, Kristin Lauter

CHES 2013

Motivation - I

	DH	ECDH
Group	$(\mathbf{F}_{p_1}^*, \times)$	$(E(\mathbf{F}_{p_2}), +)$

Security level (bits)	$\log_2 p_1$	$\log_2 p_2$	Ratio DH cost : EC cost
128	3072	256	10:1
192	7680	384	32:1
256	15360	521	64:1

Source: NSA – The case for Elliptic Curve Cryptography
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sequence of point additions and point doublings

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- See the Explicit-Formulas Database

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Reduce the **number** of group operations

- Different algorithms to compute the scalar multiplication
- Reduce the number of point additions:
  e.g. use large window sizes
- Reduce the number of point doublings:
  e.g. scalar decomposition

Reducing the Number of Point Doublings

- *d*-dimensional scalar decomposition
- Decompose a scalar *k* into *d* “mini-scalars” $k_i \approx \sqrt[d]{k}$
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$$[k]P = \sum_{i=0}^{d-1} [k_i \lambda^i]\, P = [k_0]P + [k_1]([\lambda]P) + \cdots + [k_{d-1}]\left([\lambda^{d-1}]P\right)$$

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Precompute: $\{\emptyset, P, [\lambda]P, P + [\lambda]P\}$

Example: $d = 2$

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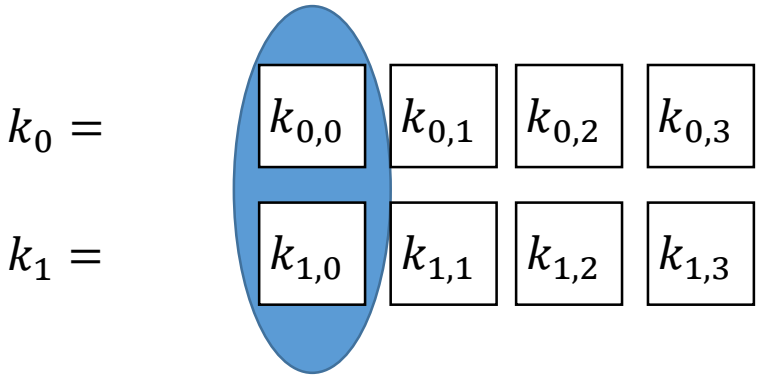

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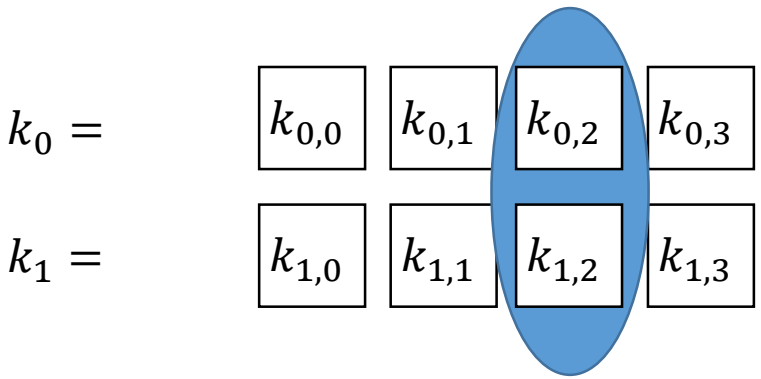

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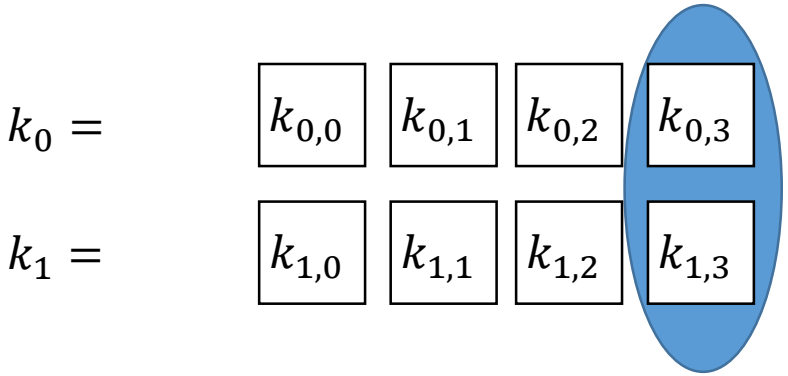

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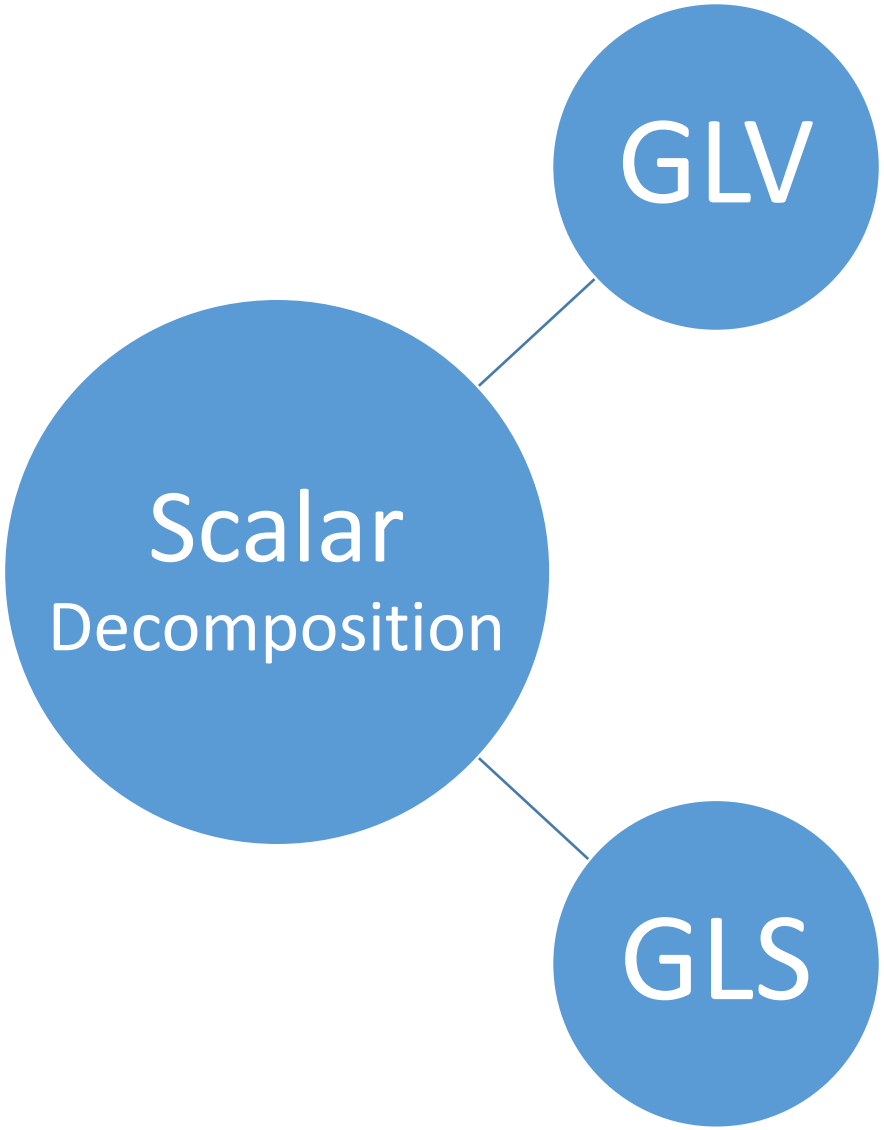
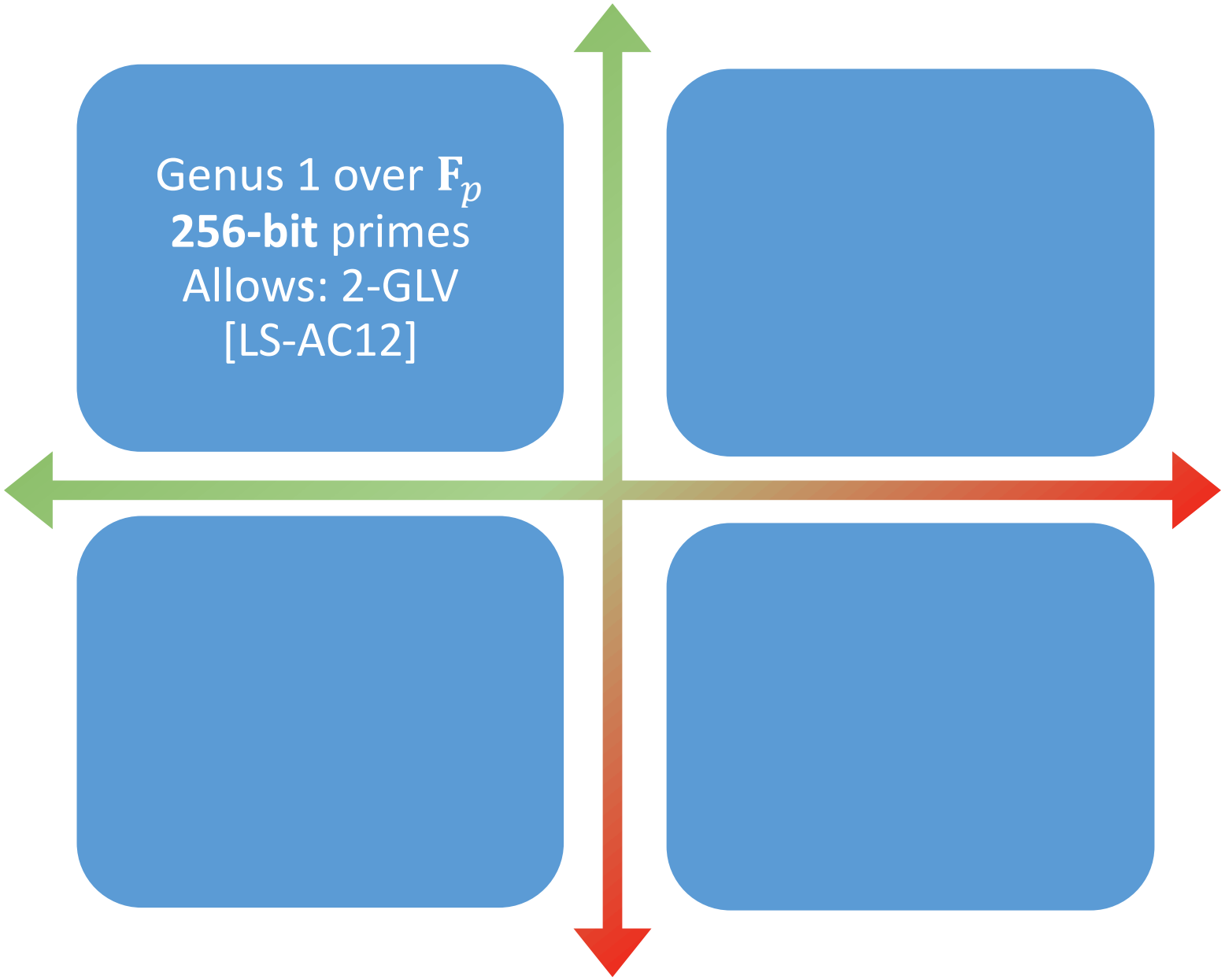


- Gallant, Lambert, Vanstone [GLV-C01]
- Use non-trivial endomorphism
- Larger endomorphism ring means larger dimensional scalar decomposition

- Galbraith, Lin, Scott [GLS-JoC11]
- Independent of the endomorphism
- Works for E($\mathbf{F}_{p^m}$) with $m > 1$

Overview of GLV/GLS techniques – 128-bit security



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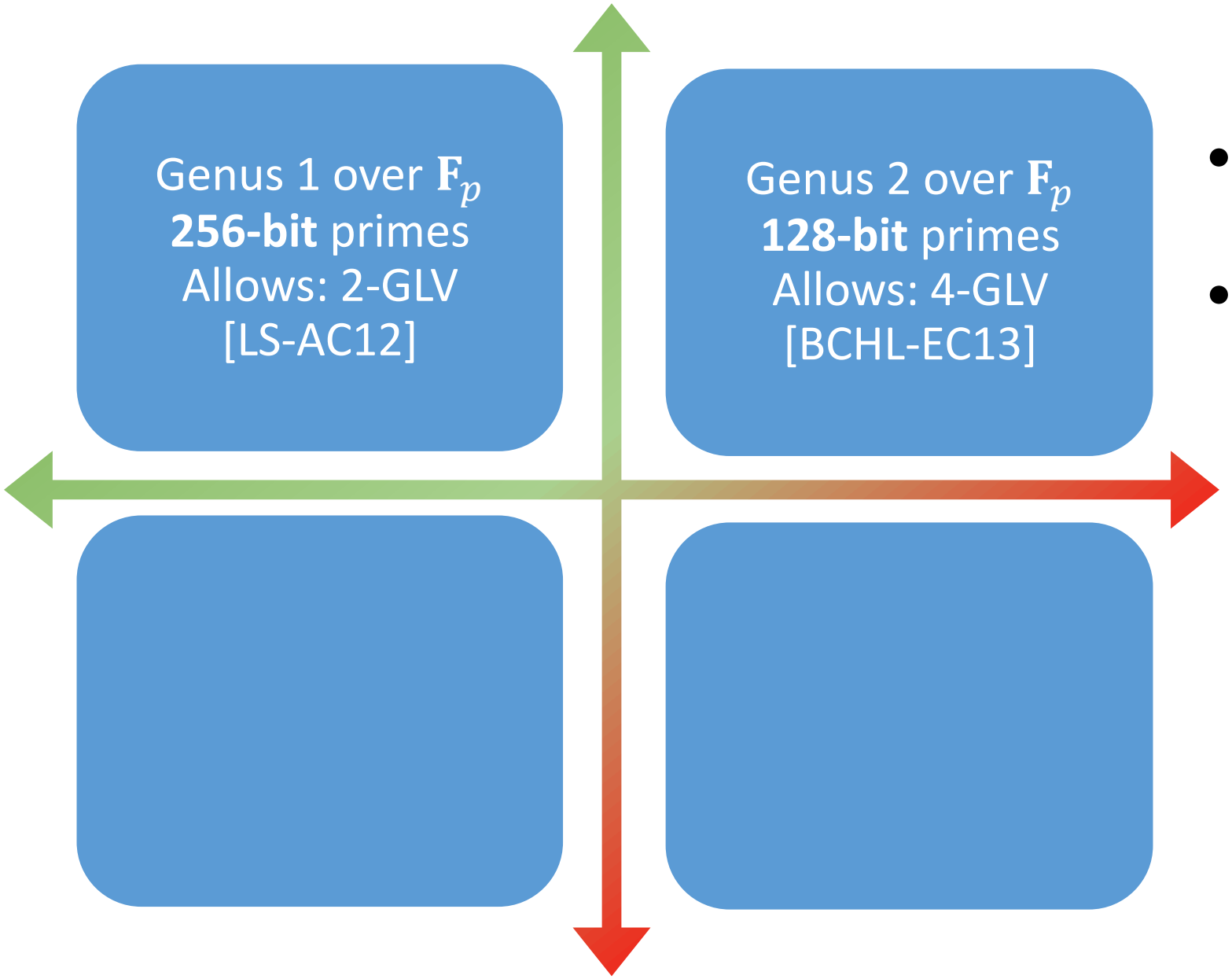

- Larger genus means larger endomorphism
- But genus > 3 is considered insecure

$$\#E(\mathbf{F}_{p_1}) \approx \#\mathrm{Jac}_C(\mathbf{F}_{p_2}) \text{ with } \log_2(p_1) \approx {}^1\!/_2 \log_2(p_2)$$

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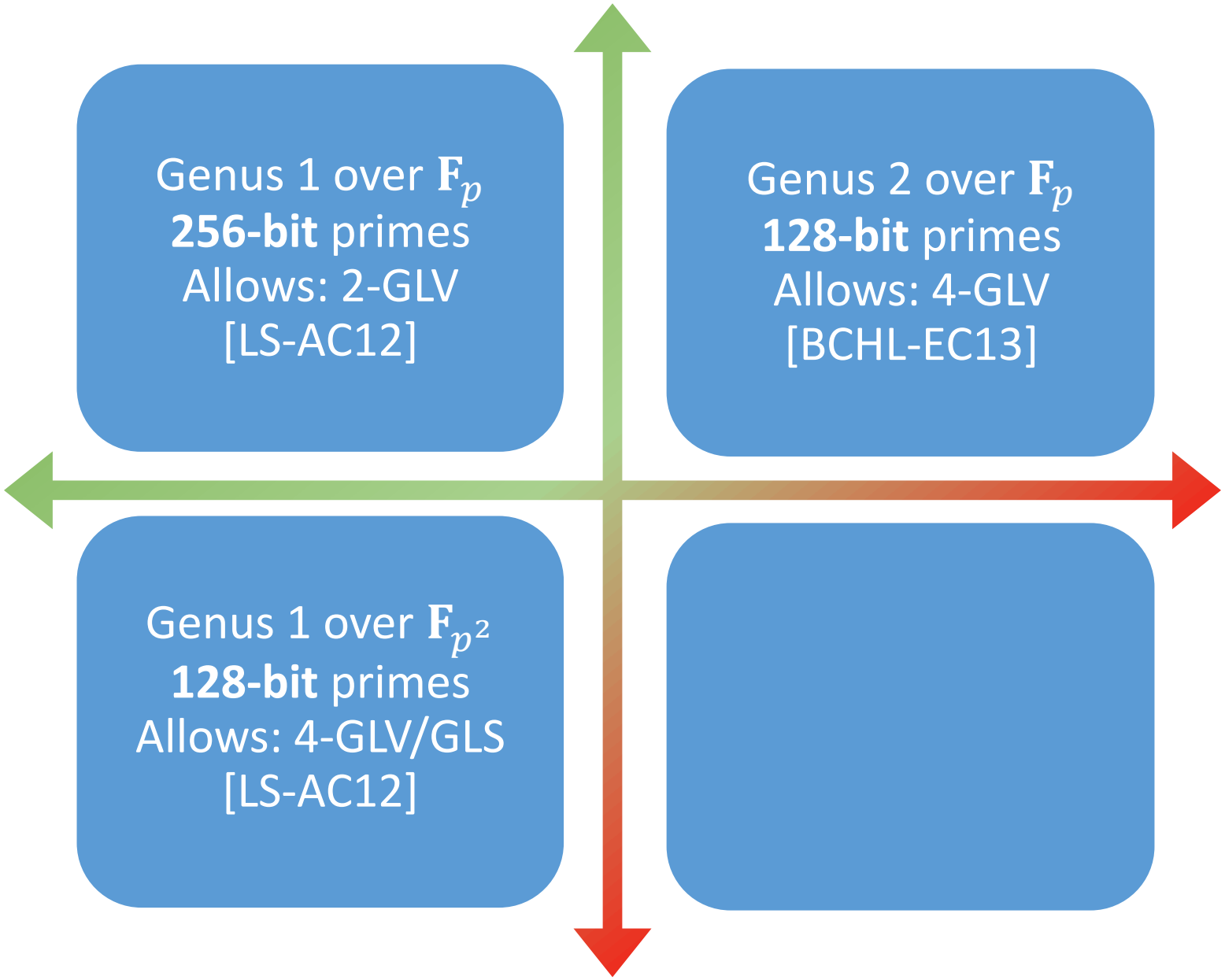

$$\#E(\mathbf{F}_{p_1}) \approx \#E\left(\mathbf{F}_{p_2^2}\right) \text{ with}$$
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- When m is too large subexponential attacks

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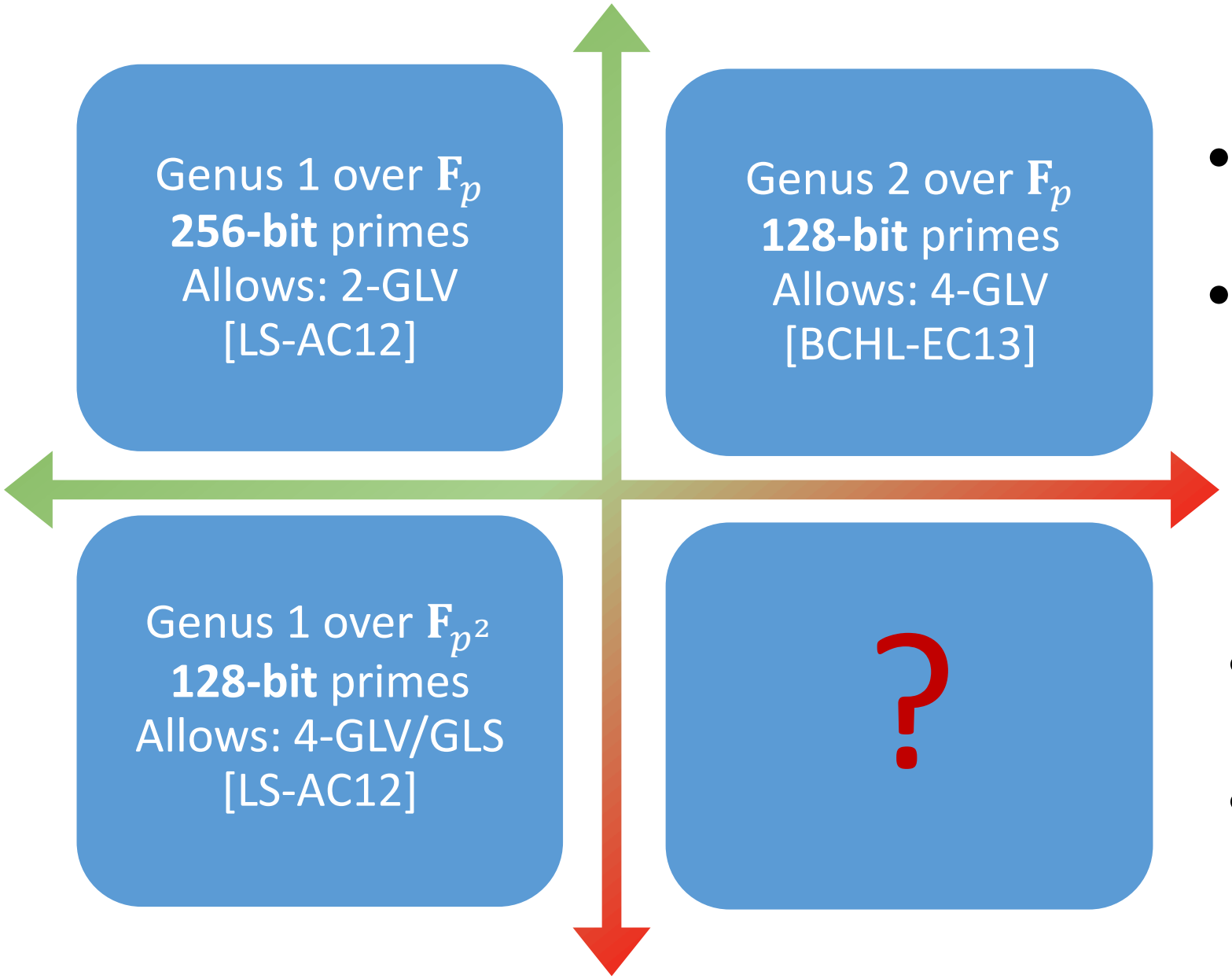

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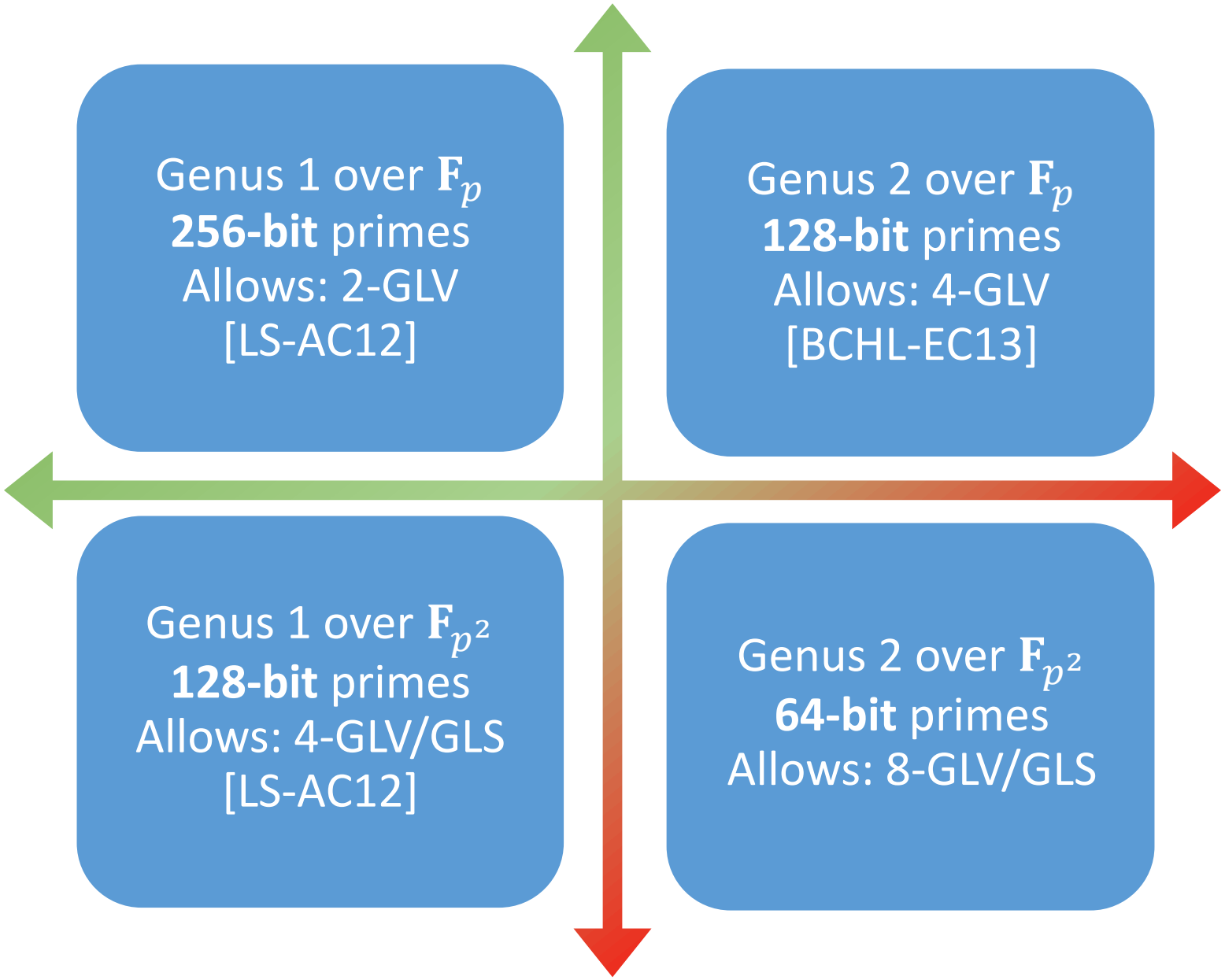

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Fits in a single word on high-end servers

Ideal candidate for embedded platforms

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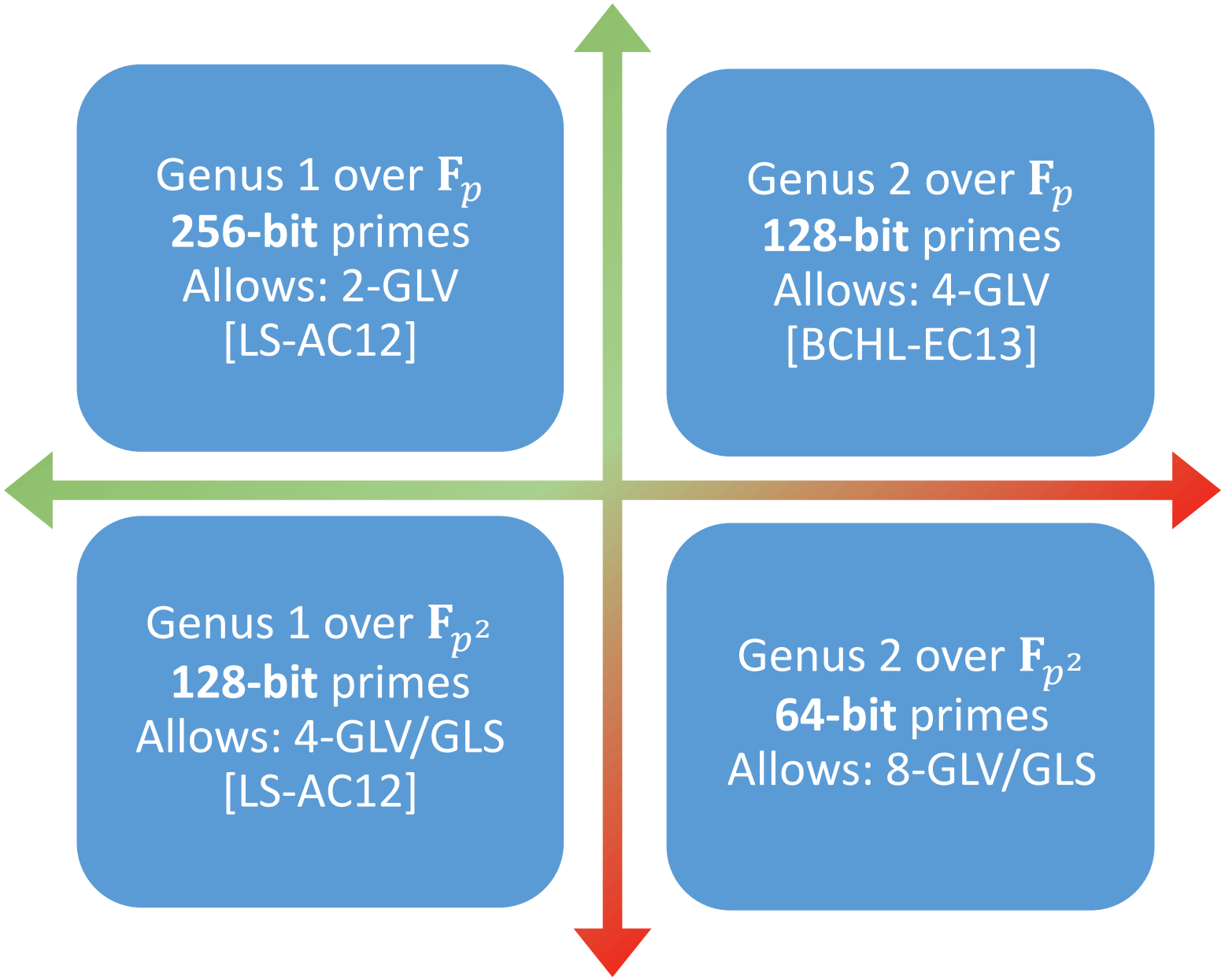

What about security?

How to efficiently use 8-GLV/GLS?

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[BCHL-EC13] uses Mersenne prime: $2^{127} - 1$, we try: $2^{61} - 1$
Also the “NIST-like” prime: $2^{64} - 2285$

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Interleaved radix-2^b Montgomery multiplication

$$C \equiv A \cdot B \cdot 2^{-bn} \bmod p, \mu = -p^{-1} \bmod 2^b, A = \sum_{i=0}^{n-1} a_i 2^{bi}$$

for $i = 0$ to $n - 1$ do

$$C = C + a_i \cdot B$$
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$= 2^{32}(2^{29}-0)-1$

Interleaved radix-2^b Montgomery multiplication

Example: $2^b(2^{\tilde{b}}-c)-1$

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Generic Attack: Pollard rho

- [Pollard-MoC78]
- $\sqrt{(\pi r)/(2\#\mathrm{Aut})}$, where $\#\mathrm{Aut} \geq 2$
- For the GLV/GLS curves $\#\mathrm{Aut} = 10$

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- [P. Gaudry. J. Symb. Comp. 09], [K. Nagao. ANTS 10]

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Advantage
Smaller scalars

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8-Dimensional GLV/GLS

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- ψ: $\mathrm{Jac}(C) \to \mathrm{Jac}(C)$, $\psi(D) = [\lambda]D$, for $0 < \lambda < r$
- Decompose the scalar using [Park,Jeong,Lim-EC02]

Lookup table: $L[i] = \sum_{\ell=0}^{7} \left(\left\lfloor \frac{i}{2^\ell} \right\rfloor \bmod 2 \right) \cdot D_\ell$ for $0 \le i < 2^8$

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Constant time
$32\mathrm{D} + 7\psi$
$(22 + 2 \times 32)\mathrm{A}$

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Non-constant time
$32\mathrm{D} + 16\psi$
$(13 + 2 \times 32)\mathrm{A}$

Performance Results – x86

Platform: Intel Core i7-3520M Ivy Bridge (2893.484 MHz), hyperthreading turned off and over-clocking ("turbo boost") disabled

We didn't aim for record-performance.
There is room for improvement!

Reference	(g, K)	CT	Bit sec	10^3 cycles
NIST-p224	$(1, \mathbf{F}_p)$		112	302
[B-PKC06] curve25519	$(1, \mathbf{F}_p)$		126	182
[FPH-ep13] 4-GLV/GLS	$(1, \mathbf{F}_{p^2})$		125	92
[BCHL-EC13] Kummer	$(2, \mathbf{F}_p)$		125	117
[BCHL-EC13] 4-GLV	$(2, \mathbf{F}_p)$		125	156
$2^{61} - 1$, Kummer	$(2, \mathbf{F}_{p^2})$		103	108
$2^{61} - 1$, 8-GLV/GLS	$(2, \mathbf{F}_{p^2})$		105	100 (88)

- 3x faster than NIST
- Slightly less secure

Performance Results – x86

Platform: Intel Core i7-3520M Ivy Bridge (2893.484 MHz), hyperthreading turned off and over-clocking ("turbo boost") disabled

We didn't aim for record-performance.
There is room for improvement!

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[BCHL-EC13] 4-GLV	$(2, \mathbf{F}_p)$		125	156
$2^{61} - 1$, Kummer	$(2, \mathbf{F}_{p^2})$		103	108
$2^{61} - 1$, 8-GLV/GLS	$(2, \mathbf{F}_{p^2})$		105	100 (88)

- Similar performance
- 105 vs 126 bits security

Performance Results – ARM

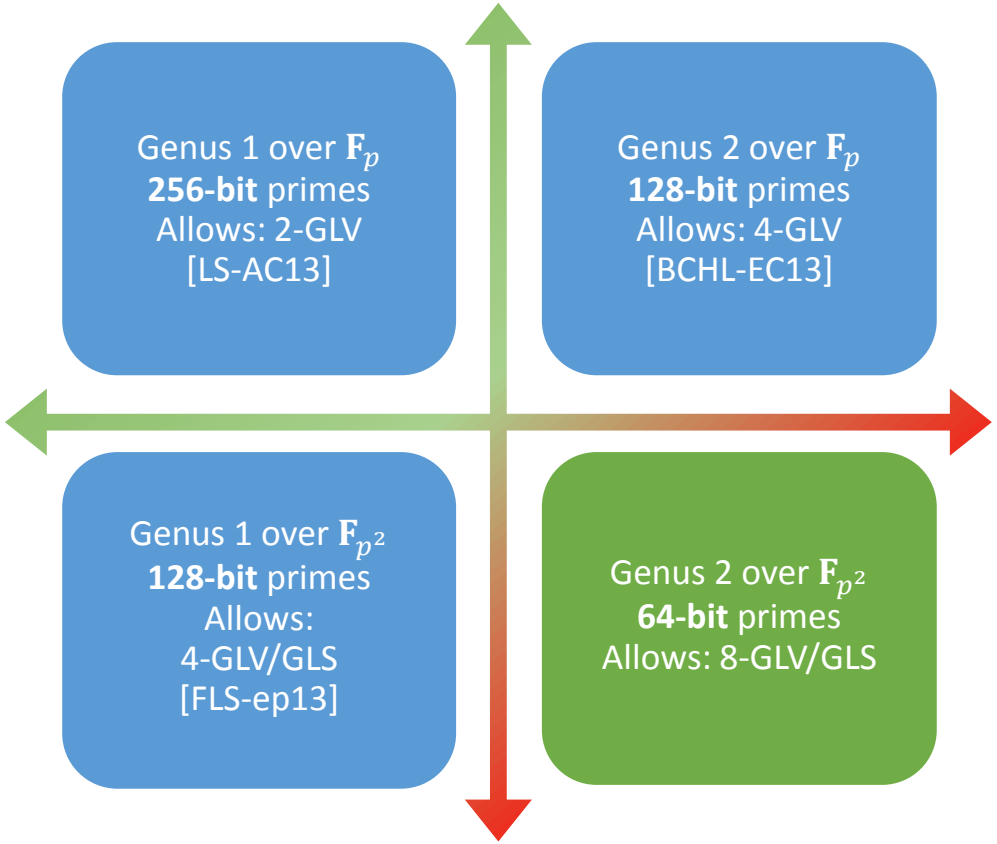
Platform: BeagleBoard-xM (1 GHz Cortex-A8 ARM core)
Use the Montgomery-friendly arithmetic

We didn't aim for record-performance.
There is room for improvement!

- Order of magnitude faster than NIST-p224
- Our performance in the same ballpark
- Lower security level

	Platform	(g, K)	CT	Bit sec	10^3 cycles
[MTS-SASP11] NIST-p224	Cortex-A8	$(1, \mathbf{F}_p)$		112	7805
[BS-CHES12] curve25519	Cortex-A8 w NEON	$(1, \mathbf{F}_p)$		126	527
[FPH-ep13] 4-GLV/GLS	Cortex-A9	$(1, \mathbf{F}_{p^2})$		125	417
[H-ep12] twisted Edwards	Cortex-A9	$(1, \mathbf{F}_p)$		125	616
$2^{61} - 1$, Kummer	Cortex-A8	$(2, \mathbf{F}_{p^2})$		103	767
$2^{61} - 1$, 8-GLV/GLS	Cortex-A8	$(2, \mathbf{F}_{p^2})$		105	617 (576)

Conclusions



- ✓ Genus 2 over $\mathbf{F}_{p^2}$ allows to work with 64-bit primes
- ✓ Interesting for both high-end 64-bit servers and embedded 32-bit devices
- ✓ Precomputing the lookup table for 8-GLV/GLS is more involved than for 2- and 4-GLV/GLS
- ✓ Although faster attacks exist, still provides sufficient security

See our full paper:
Cryptology ePrint Archive: **Report 2013/146**
The x86 implementations have been submitted to eBACS