Elliptic Curve Cryptography in Practice

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Joint work with
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Microsoft Research
Elliptic Curves in Practice – An Incomplete Overview

1933: Hasse, estimate of the number of points on an elliptic curve
\[ |\#E(F_p) - (p + 1)| \leq 2\sqrt{p} \]

1985: Schoof, deterministic polynomial time algorithm for counting points on elliptic curves
1985-1987: Lenstra Jr., elliptic curves can be used to factor integers
Miller & Koblitz, elliptic curves can be used to instantiate public-key cryptography

2000: Standard for ECC by Certicom
2006: NIST standard for ECDSA
2006: RFC 4492, ECC in Transport Layer Security (TLS)
2009: RFC 5656, ECC in Secure Shell (SSH)
2009: Nakamoto, Bitcoin

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Questions

- What is the current state of existing elliptic curve deployments in several different applications?
- Can we find problems that might signal the presence of cryptographic vulnerabilities in ECC?
Scan the complete public IPv4 space (October 2013) for SSH host keys (port 22)

Cipher suite responses:
12 114 534

- ECDSA: 1 249 273 (10.3%)
- ECDH: 1 674 700 (13.8%)
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- 1 672 458 (99.8%) supported ecdh-sha2-nistp{256,384,521}
- 25 (0.001%) supported ecdh-sha2-nistp{521,384,256}

Client offered only EC cipher suites
- 458 689 DSA public key responses
- 29 648 RSA public key responses
- 7 935 empty key responses

Hosts included several kinds of routers and embedded devices

Huawei and Mikrotik.
Repeated keys (cloud hosting providers)

- shared SSH infrastructure that is accessible via multiple IP address
- mistake during VM deployment

Digital Ocean: “The SSH host keys for some Ubuntu-based systems could have been duplicated by DigitalOcean’s snapshot and creation process.”

5614 hosts served the public key from Digital Ocean’s setup guide

- Default keys present in the hardware or poor entropy on boot

Juniper Web Device Manager, the Juniper FemtoAP, and ZTE Wireless Controller

Scan the complete public IPv4 space (October 2013) for SSH host keys (port 22)

Cipher suite responses: 12,114,534

- 1,672,458 (99.8%) supported ecdh-sha2-nistp{256,384,521}
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Hosts included several kinds of routers and embedded devices

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Scan the complete public IPv4 space for TLS cipher suits (port 443)

Total hosts: 30.2M

ECDH(E) 2.2M (7.2%)

- 98% nistp256
- 80% nistp384
- 17% nistp521

1.7 million hosts supported > 1 curve
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- 354,767 hosts
  - secp{256,384,521}r1
- 190 hosts
  - secp{521,384,256}r1

Hosts prefer lower computation and bandwidth costs over increased security

1.7 million hosts supported > 1 curve
Many duplicated keys are from small set of subnets, most likely nothing wrong: single shared host, but

- **A single key** presented by 2000 hosts
- 1800 Netasq devices presented the same NISTp256 key for ECDHE key exchange

**buying this device allows one to decrypt traffic from all other devices**

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**1.7 million** hosts supported > 1 curve
From asymmetric crypto point of view Bitcoin relies exclusively on ECDSA

**August 2013:** Bitcoin block chain (#252 450)
- Extracted **22M** transactions (26GB plaintext file)
- **46M** signatures
- **46M** ECDSA keys
  - **15.3M** unique

**March 2014:** > **12.4 million** bitcoins in circulation
estimated value: > **8.4 billion** USD
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Interesting choice:
not NIST P-256 but “special” sec256k1

secp256k1: $p \equiv 1 \pmod{6}$, there exists $\zeta \in \mathbb{F}_p$, such that $\zeta^6 = 1$

$$
\psi : E \to E, (x, y) \to (\zeta x, -y)
$$

Fast scalar multiplication $\psi(P) = \lambda P$
for an integer $\lambda^6 \equiv 1 \pmod{n}$

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R. P. Gallant, R. J. Lambert, and S. A. Vanstone. Faster point multiplication on elliptic curves with efficient endomorphisms. CRYPTO 2001
Elliptic Curve Digital Signatures \((d, Q, m)\)

\[ k \in \mathbb{F}_n^\times, \quad kG = (x, y), \quad r = x \mod n \]

\[ s = k^{-1}(\text{Hash}(m) + dr) \mod n, \quad \text{Signature: } (r, s) \]

We require \( r \neq 0 \neq s \) and \( k \) is a per-message secret since

if \((r, s_1)\) and \((r, s_2)\) then \( k \equiv (s_2 - s_1)^{-1}(e_1 - e_2) \mod n \)

\[ d \equiv r^{-1}(ks - \text{Hash}(m)) \mod n \]
We looked for duplicated nonces in the signatures. 158 unique public keys had used the same signature nonce $r$ value in more than one signature → making it possible to compute these users’ private keys.

Currently 0.00031217 BTC = 0.21 USD left on these accounts.

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Address: 1HKywxil4JziqXrzLKhmb6a74ma6kxbSDj

March to October 2013: 59 BTC \(\approx\) 40076 USD has been stolen from 10 of these addresses.
Elliptic Curve Digital Signatures \((d, Q, m)\)
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Possible cause

**Poor entropy?** At least 3 keys are known to be generated by implementations with Javascript’s RNG problem
Conclusions

✓ **ECC is well-deployed and used in practice**

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✓ ECC is not immune to insufficient entropy and software bugs

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<td>• We found many instances of repeated public SSH and TLS keys</td>
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<td>• Bitcoin: there are many signatures sharing ephemeral nonces</td>
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<td>This lead to the theft of at least 59 BTC</td>
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See our paper in Financial Cryptography and Data Security 2014 and on eprint:
http://eprint.iacr.org/2013/734